SOUND

A TEXTBOOK OF PHYSICS

By R. C. BROWN, B.Sc., Ph.D.

Volume 1 MECHANICS AND PROPERTIES OF MATTER

Volume 2 HEAT

Volume 3 SOUND

Volume 4 LIGHT *In the press*

Volume 5 MAGNETISM AND ELECTRICITY *In preparation*

Complete Edition *In preparation*

SOUND

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With Diagrams

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PREFACE

The greater part of what an Intermediate or Advanced-Level student is expected to know about Sound concerns the mechanics of vibrations and waves. Much of this volume is therefore devoted to these topics, and the treatment is similar to that adopted in Volume 1. I have included a fuller presentation of forced and resonant vibrations than is usually found in a book of this standard in the hope that it will be found more informative than the "child in a swing" treatment and will help to dispel the notion that maximum amplitude always occurs when the applied and natural frequencies are equal.

In dealing with the vibrations of such systems as stretched strings and gas columns I have freely used the idea of Fourier analysis but not, of course, its mathematical treatment.

The chapter on the Subjective Effects of Sound is intended to give an adequate idea of the relation between the mechanical properties of vibrations and their subjective consequences without going into details of musical scales and theories of audition.

It is obviously impossible at this comparatively elementary stage to give a quantitative treatment of such matters as the recording and reproduction of sound, and a merely descriptive account is, I think, rather out of place in a Physics course and may add little to the student's general knowledge of the principles of the gramophone, sound film, etc. I have therefore made little reference to these applications of acoustics.

I wish to thank the following examining bodies for having given permission for the inclusion of questions from past papers:—

The Delegates of Local Examinations, Oxford (O.).
The University of Cambridge Local Examinations Syndicate (C.).
The University of London (L.).
The Joint Matriculation Board (J.M.B.).

The various examinations are indicated in the text by the above initials in conjunction with letters according to the following scheme:—

Intermediate—I.
Higher School Certificate—H.S.
Advanced Level—A.
Scholarship—Schol.
First Medical—Med.

In transcribing the questions I have usually refrained from modifying the various modes of writing the names of physical units. The reader is thereby enabled to become acquainted with other common methods of expressing units in addition to the "index" convention which is used in the text.

R. C. Brown.

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VOLUME III

SOUND

Chapter XXXIV

VIBRATORY MOTION AND WAVES

1. INTRODUCTION

The Nature and Cause of Sound.—We give the name **sound** to the sensation which we perceive with our ears, and the branch of Physics which bears this title deals with the mechanism of the production and propagation of sound.

Every sound is produced by the vibration of the body from which it originates. This fact can easily be verified by touching a sounding bell, violin string, etc. The vibrations can be felt with a light touch of the finger, and the sound ceases when the vibrations are stopped by a firmer grip. A vibrating source of sound sets the air (or other medium) in its neighbourhood into oscillation and causes sound waves to travel outwards from it. On reaching the ear, the waves cause the ear-drum to vibrate in a way which reproduces the oscillations of the source.

It can be shown that a material medium is necessary for the transmission of sound waves by placing a source of sound, such as an electric bell or buzzer, in a chamber from which the air can be removed by a pump. As the air is pumped out, the sound received by the ear becomes fainter, and the bell is eventually inaudible although it can still be seen to be in operation. It is necessary that the bell should not be in contact with the walls or floor of the chamber, otherwise the sound will be transmitted to the atmosphere through these.

Sounds may roughly be divided into two categories—**noises** and **musical notes.** The former are produced when the motion of the sounding body is irregular in the sense that it does not repeat itself rhythmically with any definite frequency, or if it does do so the frequency of repetition is less than about 30 per sec. When a vibrating body performs a type of motion which is regularly repeated with a higher frequency than this, the hearer becomes aware of a sound of definite pitch, which may or may not be pleasing to the ear according to the nature of the motion performed during each repetition. If the edge of a thin sheet of metal or other material is held against the teeth of a large cog-wheel and the wheel is then rotated with increasing speed, the sound made by the

knocking of the teeth on the plate is at first only a series of separately distinguishable taps. As the frequency of the taps is raised by the increasing speed of the wheel, a note of definite pitch is heard which evidently gets higher as the frequency rises. If the speed of rotation of the wheel can be measured so that the frequency of the taps can be deduced, this simple device may be used to investigate how the pitch of a note varies with the frequency of the source and to show, for instance, that doubling the frequency raises the pitch by an octave.

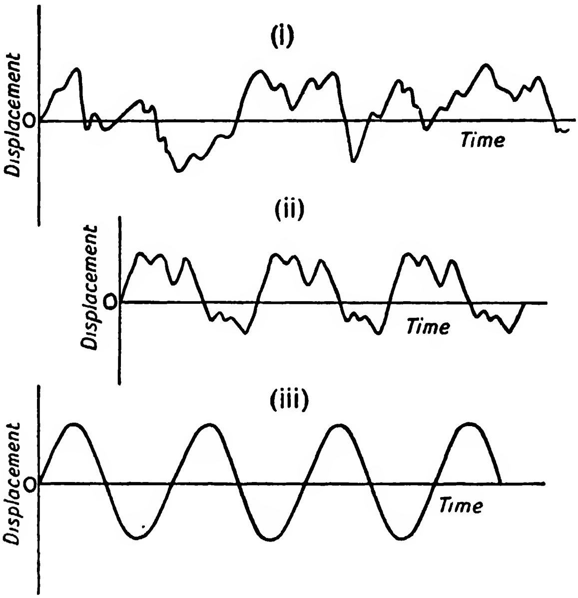


FIG. 399

The distinction between a noise and a musical note is illustrated in Fig. 399, in which (i) is a typical displacement-time graph of a particle of air when a noise is being transmitted. It will be noticed that the graph exhibits no periodic repetition. Fig. 399 (ii) is the sort of displacement-time graph performed by an air particle in the neighbourhood of a stringed instrument, and the simpler graph in (iii) is of the type produced by a tuning-fork. It is the shape of the repeated pattern—the **wave-form** as it is called—which gives the note its individual **quality** or **timbre.** This is the property of a musical note which distinguishes it from another emitted by a different instrument even when their pitches are identical.

It will be noticed in Fig. 399 (iii) that the shape of the displacement-time curve for a tuning-fork is that of a sine curve. This indicates that the prongs of the fork are executing simple harmonic motion. We shall often examine theoretically the behaviour of vibrating bodies and

sound waves on the assumption that the motion they perform is simple harmonic. It may appear at first sight that this is an unjustifiable simplification since a sine wave-form is much less common than more complicated forms. It can be shown however that, no matter what may be the nature of a vibratory motion which repeats itself regularly, it can be represented as the resultant or combination of a large number of simple harmonic motions of suitably chosen amplitudes having frequencies which are integral multiples of (*i.e.* 1, 2, 3 . . . times) the frequency with which the motion repeats itself. This principle, which is known as **Fourier's theorem,** is illustrated in Fig. 400. Graph (i) is a hypothetical displacement-time graph which appears at first sight to be very far removed from a sine graph inasmuch as it consists of horizontal and vertical *straight* lines.

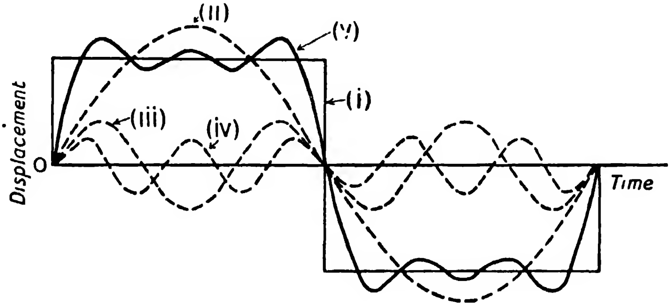


FIG. 400

However, when the three sine waves (ii), (iii) and (iv) are superposed on each other, that is to say the separate simultaneous displacements due to each are added algebraically, curve (v) is obtained, which approximates to the original curve (i). When the number of component sine waves is increased indefinitely it is possible to reproduce the original graph perfectly. The separate sine vibrations which together make up a given wave-form are called **harmonics,** the one of lowest frequency being the first harmonic of the series. In any particular case all the possible harmonics may not be present. For example, the analysis of the note given out by a closed organ pipe (page 664) does not contain the harmonics of frequencies 2, 4, 6 . . . times the first. This is also the case in Fig. 400.

2. WAVE MOTION

The Mechanism of Wave Propagation.—It is possible to discuss the process of wave propagation by reference to a system such as that depicted in Fig. 401 (i). A number of particles ABCD . . . lie in a straight line and are joined by springs which are considered to have no mass. Let the particles be free to move only in a direction at right angles to the line joining them. Let the first particle A be given an upward displacement at right angles to the line of particles and then let it be brought

back to its original position. Fig. 401 (ii) shows the commencement of this motion. Evidently as soon as A begins to move, the spring joining it to B exerts on B a force which has an upward component so that B begins to follow A. This causes C and some of the following particles to begin to move by the same mechanism.

If the particle B possesses mass, a force is necessary in order to set it in motion, and therefore its motion does not begin until A has gone a small distance and the connecting spring is slightly tilted. Thus B lags behind A while they both move upward. When A comes to rest at the extreme of its displacement and immediately begins to travel downwards (Fig. 401 (iii)), the force which it was exerting on B through the spring diminishes

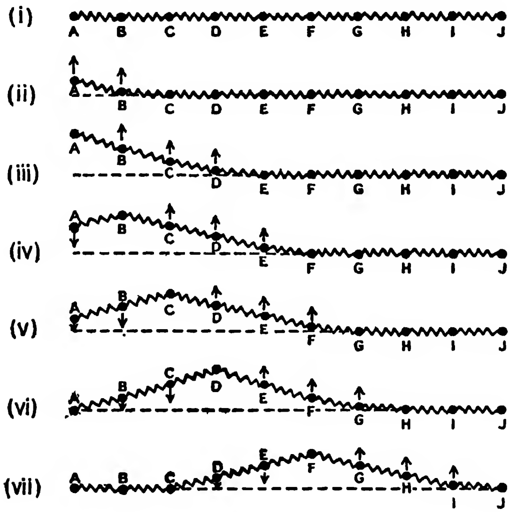


FIG. 401

to zero and then increases in a downward direction. At this stage the momentum which B possesses by virtue of its mass and velocity causes it to continue to move upwards until it is brought to rest by the downward forces due to the stretched springs connecting it to A and to C. The particle B is now the most displaced particle. This stage of the motion is shown in Fig. 401 (iv). The particle B is then pulled downwards by A and begins to descend, still lagging behind A, while C continues to move upwards until it takes its turn as the most displaced particle before descending (Fig. 401 (v)). Eventually, after A has regained its original position, the condition of the particles is as shown in (vi) and later as in (vii), where a "hump" or disturbance of constant shape is shown moving along the line of particles. The particles themselves move only at right angles to the line. The moving disturbance is part of a **wave.**

The process just described can be demonstrated in practice by two persons grasping the ends of a fairly long rope and holding it taut. If one person then shakes his end of the rope sharply, the disturbance so created will travel along the rope and be felt by the other person after a finite interval of time. It is evident that if, after regaining its original position, the particle A in Fig. 401 immediately makes a similar excursion downwards and back again, the complete oscillation which it has now made will give rise to a disturbance of the kind shown in Fig. 402. This

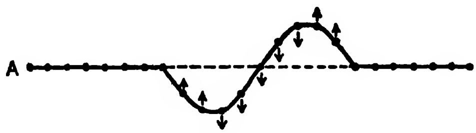


FIG. 402

will travel along the line of particles. Furthermore, a continuous wave will be generated if the oscillations of A are maintained (Fig. 403).

When a wave such as that shown in Fig. 403 is passing over the line of particles, each particle is performing oscillations which are exact replicas of the oscillation of A. At any given instant, however, the phase of the oscillation varies from one particle to the next.

An instantaneous picture of a wave such as Fig. 403 can be regarded

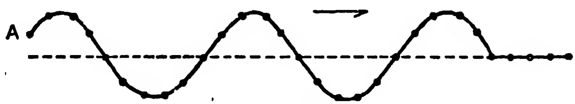


FIG. 403

as a graph connecting the instantaneous displacement of each particle plotted vertically against its undisturbed position plotted horizontally. This graph is known as a **displacement curve,** and its shape is governed by the nature of the oscillation of the first particle A. Suppose, for example, that A moves rapidly when it is travelling upwards and slowly when it is descending. Only a few particles will be displaced by the time A has reached the top of its swing so that the initial part of the disturbance will be steep (Fig. 404), whereas the second part will be less steep because the maximum displacement will travel a considerable distance before A regains its undisturbed position. If the speed with which the disturbance is propagated is independent of its shape, we can picture the wave (*i.e.* the displacement curve) as being traced by the oscillating particle A on a strip of paper which is moving from left to right with the speed of propagation. By considering this

FIG. 404

system it becomes clear that the displacement curve has the same shape as a graph of the displacement of A—or any one of the particles—against time.

It is evident from the foregoing discussion that for the propagation of a wave through a medium it is necessary that when a particle is displaced it should exert a force on a neighbouring particle and so hand on its own displacement. It may be noticed that this implies (by Newton's third law) that the displaced particle suffers a restoring force tending to send it back to its original position. In Fig. 401 the displacement of A causes B to be pulled upwards by the spring connecting the two particles, and as soon as B moves, C is pulled upwards and so on. The system considered here is, of course, a simplified model. When a wave is transmitted through a piece of material the property which is called into play is the **elasticity** of the substance—the property by virtue of which two adjacent particles exert a force on each other when their relative positions are changed. There are several types of wave corresponding to the various possible types of deformation, *viz.* shearing, stretching and compression, the last being the only one possible in fluids.

A disturbance is propagated along a rope, however, by virtue of the tension in the rope rather than of its elasticity, and when waves travel on the surface of water the force involved is due to the hydrostatic pressure caused by gravity.

We may note at this stage that we should expect the speed with which the disturbance is propagated to be large when, for a given relative displacement, the force between two particles is large. Thus if the springs in Fig. 401 are strong or are already in a state of large tension before the particle A is moved, the force exerted on B is large for a given displacement of A. The particle B will therefore follow more closely behind A than it would if the spring connecting them was weak or slack. For this reason not only does the disturbance travel faster when the springs are strong, but (what is really the same thing) the length of the disturbed region or "hump" is greater. When waves are transmitted through solids or fluids the elasticity of the material plays the part of the springs, and the speed of propagation is high and waves of a given frequency are long when the material has a high modulus of elasticity so that relative displacement of its particles calls into play a large force.

We can also discuss qualitatively at this stage the part played in wave propagation by the masses of the particles in Fig. 401. For a given displacement of the first particle A a given force acts on B, but the velocity which B acquires in a given short time is inversely proportional to its mass. Therefore when the mass is large B follows less closely behind A so that, for a given set of springs and a given rate of oscillation of A, particles of large mass would give short waves and a slow rate of propagation, while particles of small mass would give long fast waves. The foregoing arguments lead us to expect that when waves are propagated in a continuous

medium, the expression for their speed would contain the appropriate modulus of elasticity in the numerator and the density of the material in the denominator. A more quantitative investigation is given on page 569.

Definitions.—The **speed** (often called the "velocity") of a wave is the speed with which its outline is travelling in the direction of the wave. In order to be more precise we may name a particular point on the wave such as a crest, trough or point of no displacement. We shall denote the speed of waves by c.

The **wave-length** (λ) is the shortest distance between any two crests, troughs or points of no displacement or, indeed, between any two particles which are in the same phase of their oscillation, *e.g.* D and D′ in Fig. 405.

The maximum displacement from its mean position which any given particle suffers is the **amplitude** of the wave at the position of that particle. This is marked as a in Fig. 405.

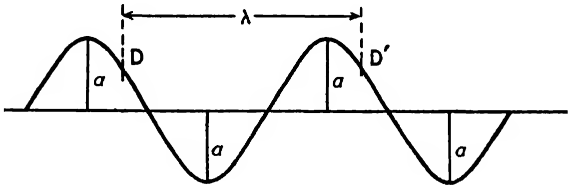


FIG. 405

Suppose that T is the time period of the oscillation of each particle over which a wave of wave-length λ and speed c is travelling. It is clear that after a complete oscillation of every particle a given crest or trough has moved through a distance λ into the position previously occupied by the adjacent crest or trough. The outline of the wave has therefore travelled a distance λ in a time T, and the speed of the wave is given by λ/T or $n\lambda$, where n is the frequency of oscillation of every particle and is equal to $1/T$. This is a very important relationship which is true of all types of wave.

So far, we have discussed waves which are known as **transverse** because the particles over which they pass vibrate at right angles to the direction of propagation. In longitudinal waves the directions of oscillation and propagation are identical (page 559).

We shall discuss later (page 563) a type of wave which is described as **stationary** in contrast to the waves dealt with above, which are known as **progressive** on account of their motion through the medium.

Sinusoidal Waves.—It has already been mentioned that the behaviour of waves can often be usefully studied by supposing that the particles over which the wave is passing are performing S.H.M. Let a wave pass over a line of equally spaced particles, and let it be of such a type that its

passage causes all the particles to perform S.H.M. with the same frequency and amplitude. One of the characteristics of wave motion is, as we have seen, a phase difference between consecutive particles: each particle lags behind its neighbour on the side nearer to the source. Let there be a *constant* phase difference between consecutive particles. We can now construct graphically the outline of the wave (the displacement curve) at any instant.

On the left of Fig. 406 is a circle by which the simple harmonic motions are generated. Let the angles between adjacent radii OP_0, OP_1, OP_2, etc. be equal, and let all the radii rotate in an anticlockwise direction with constant angular velocity. The feet of the perpendiculars from P_0, P_1, P_2, etc. on to AOB will then perform S.H.M. between A and B, the phase of each point differing from that of its immediate neighbours by the same amount on account of the uniform angular separation of the radii. Let

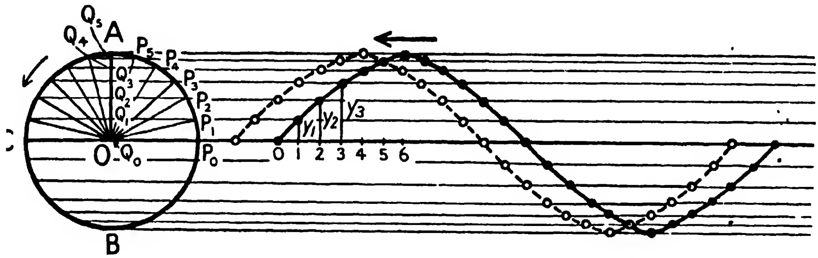


FIG. 406

the points 0, 1, 2, 3, etc. represent the undisturbed positions of the particles over which the wave is passing, and let the motion of particle 0 be the same as that of the point Q_0, that of 1 the same as Q_1 and so on. The instantaneous position of each particle can then be plotted by producing the lines Q_0P_0, Q_1P_1, etc. and marking their intersections with vertical lines through the points 1, 2, 3 . . . Thus the particle 0 has no displacement, the displacement of particle 1 is y_1, which is equal to OQ_1 and so on. The shape of the wave (the displacement curve) at the instant under consideration is the line passing through the instantaneous positions of the particles. The rotating radii in the quadrant of the circle P_0A account for the first quarter of the wave up to the first crest, while those in the quadrant AC correspond to the next quarter from the crest down to the undisplaced particle. The second half of the complete wave is provided for by the lower half of the generating circle. The complete pattern, representing one wave-length, can be repeated indefinitely.

The propagation of the wave may be examined by plotting the positions of the particles at some slightly later time. Suppose, for example, that an interval of time has elapsed in which each radius of the circle has

moved in an anticlockwise direction into the position previously occupied by the next but one. Thus, for example, OP_1 will move into the former position of OP_3, which means that particle 1 now has the same displacement as particle 3 had at the beginning of the interval and so on. Each particle is now in the position indicated by the small open circles in Fig. 406, and it is evident that the transverse motions of the particles have caused the wave to travel without changing its shape. If the radii of the circle are rotated in the opposite direction to that shown in the figure, the wave will travel from left to right.

We can now proceed to establish an equation representing the shape of the displacement curve, *i.e.* the equation connecting the displacement y of any particle with its horizontal distance x from any chosen point.

Consider a point R′ on the displacement curve (Fig. 407). It represents the displaced position of a particle whose equilibrium position is R. Let the point R be at a distance x from the point of zero displacement O, and

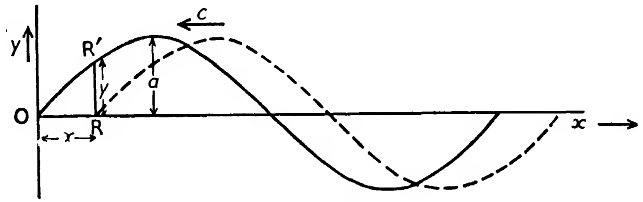


Fig. 407

let the displacement RR′ be y. The particle under consideration is performing S.H.M., and therefore its displacement is given by

$$y = a \sin \omega t$$

where a is the amplitude of the oscillation, t is the time which has elapsed since the displacement was zero,* and ω is related to the time period of the oscillation T by

$$2\pi$$

When the displacement of the particle was zero the wave, which is travelling to the left with a speed c, was in the position shown by the dotted line. It therefore follows that in the time t, during which the displacement of the particle under consideration has changed from zero to y, the wave has travelled a distance x. Therefore

$$t = \frac{x}{c} = \frac{xT}{\lambda}$$

* On page 36 (Vol. 1) the equation $y = a \cos \omega t$ is used to represent S.H.M. The sine and cosine forms are exactly equivalent except in the choice of time zero. In the cosine form the time is reckoned from one of the instants at which the particle occupies a position of maximum displacement.

so that $$\omega t = 2\pi t \quad 2\pi x$$

Thus the displacement at R′ is given by

$$y = a \sin \frac{2\pi x}{\lambda}$$

This, therefore, is the equation of the displacement curve referred to an origin at a point of no displacement and moving with the wave. It tells us nothing about the direction or speed of propagation of the wave—merely the shape of the wave as seen by an observer moving along with it. It will be noted that y is zero when $x = \lambda/2$, λ, $3\lambda/2$, 2λ, $5\lambda/2$, etc., and has its maximum value (namely a) when $x = \lambda/4$, $3\lambda/4$, $5\lambda/4$, etc.

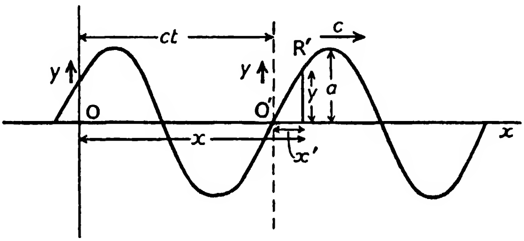


FIG. 408

In order to bring the velocity of the wave into the equation it is necessary to suppose that the observer, *i.e.* the origin from which distances are measured, is at rest. Let the wave in Fig. 408 be moving to the right with a velocity c. The origin O′ is moving with the wave so that if the co-ordinates of a point R′ on the wave with respect to axes through O′ are (x', y) we have, as above,

$$y = a \sin \frac{2\pi x'}{\lambda} \qquad (1)$$

The other origin, O, is at rest, so that O′ and the wave are moving with a velocity c with respect to O. If t is the time which has elapsed since O′ was at O, then

$$OO' = ct$$

and if x is the abscissa of R′ with respect to O, we have

$$x' = x - OO'$$
$$= x - ct$$

Substituting for x' in equation (1) we obtain

$$y = a \sin \frac{2\pi}{\lambda}(x - ct)$$

which is the equation of the wave with respect to the fixed origin O. The equation can be put in a variety of forms. Thus

$$y = a \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right), \quad \text{since } c = \frac{\lambda}{T}$$

or

$$y = a \sin 2\pi\left(\frac{x}{\lambda} - nt\right), \quad \text{since } n = \frac{1}{T}$$

or

$$y = a \sin \left(\frac{2\pi x}{\lambda} - \omega t\right), \quad \text{since } \omega = \frac{2\pi}{T}$$

For a wave of the same shape but travelling in the opposite direction, *i.e.* in the negative direction of the x axis, we can regard c as negative and write the equation

$$y = a \sin \frac{2\pi}{\lambda}(x + ct)$$

The significance of the foregoing equations is appreciated by noting the effect of supposing that x and t are constant in turn. When x is constant the value of y is periodic in time, which means that any given particle performs S.H.M., while when t is constant y varies periodically with x, and the equation gives the sinusoidal shape of the displacement curve at any instant. Both the displacement-time curve for any given particle and the shape of the wave at any given time have the same form as a graph of the sine of an angle against the angle itself. In the former case the angle is ωt or $2\pi t/T$, and increases by 2π for every increase of t by T, thus causing a repetition of the motion of the particle. Similarly, in the case of the displacement curve the angle to whose sine the displacement is proportional is $2\pi x/\lambda$, which increases by 2π whenever x increases by λ, so the curve consists of a series of waves each of length λ.

Longitudinal Waves.—We come now to a study of the waves by which sound is propagated in fluids, namely longitudinal waves. We have seen that an essential part in the transmission of waves is played by the force between two particles which comes into existence when they are displaced with respect to each other. In a transverse wave this is a shearing force because, during the passage of a wave, adjacent particles move through different distances in a direction at right angles to the line joining them. A shearing force cannot exist in a fluid, however (page 157, Vol. 1); which means that if one particle is displaced relative to another in the manner of a transverse wave the first particle does not

drag the second particle after it. On the other hand, when a particle of a fluid is moved directly towards another particle, a local increase of pressure is set up which tends to move the second particle in the same direction as the first. A longitudinal displacement of this kind can therefore be transmitted along a line of particles. The force involved is the force with which the fluid resists compression, and it is therefore the bulk modulus of elasticity which governs the rate of propagation of a longitudinal disturbance or wave.

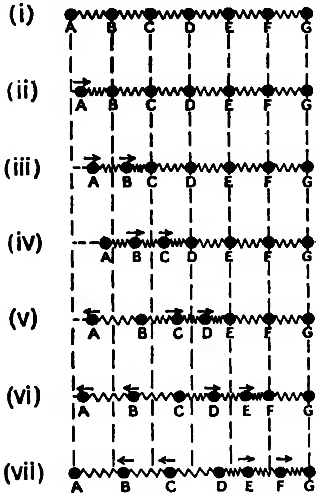


FIG. 409

Fig. 409 (i) shows a row of equally spaced undisturbed particles joined by springs. The subsequent drawings indicate the effect of moving the first particle A through half a vibration, *i.e.* towards B, and back again. In (ii) A has been moved, and the diminished distance between A and B compresses the spring between them, which causes B to begin to move to the right. The movement of B sets C in motion and so on. There is a lag between the motion of each particle and the one which precedes it because, owing to the finite masses of the particles, a finite force is necessary in order to displace them. In (iv) A has reached the end of its excursion and the other particles have increased their displacements. In (v) A has begun to return, while the momentum of B has carried it still further to the right against the action of the spring joining it to A, which is now stretched. Thus B has become the most displaced particle. The process is of the same nature as in a transverse wave, the disturbance being propagated by continuously growing in one direction (towards the right in this case) and diminishing in the other direction.

It should be noticed by reference to, say, (vi) that the particles lying immediately ahead of the most displaced particle (which is C in (vi)) are more crowded together than normally. This condition is known as a **condensation,** which in a continuous medium is a condition of high density and pressure. The particles behind the one which is most displaced are further apart than in their undisturbed state, and are in a state of **rarefaction** or low density and pressure. These conditions of abnormal density are transmitted along the line of particles. When A continues its oscillation by moving to the left of its original position, it initiates a further rarefaction which is continuous with the first. The last stage of a complete oscillation of A, *i.e.* its return to the initial condition, sends a condensation along the line of particles, and the beginning of the

next oscillation continues the condensation, which is then followed by a rarefaction and so on. Thus a longitudinal wave consisting of alternate condensations and rarefactions travels along the line.

Fig. 410 (i) is a representation of the condition at a given instant of a medium through which a longitudinal wave is travelling. The vertical lines represent layers of the medium which are equidistant when they are in their undisturbed positions which are denoted by the dots. Such a wave resembles a transverse wave in that it consists of a continuous oscillation of each particle with a phase difference between consecutive particles. It is possible to draw a displacement curve showing the displacement of every particle at a given instant, and since, in drawing a graph,

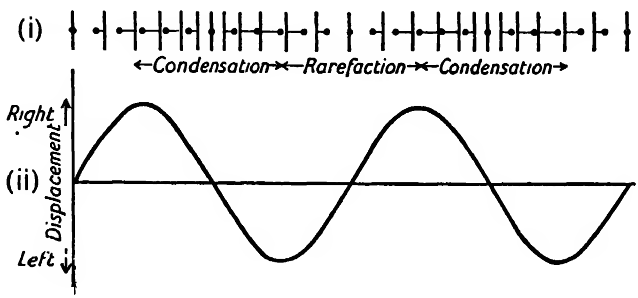


Fig. 410

we represent displacements vertically, *i.e.* at right angles to the line of particles, the curve takes the form of a transverse wave. This is illustrated in Fig. 410 (ii), in which displacements are plotted on a magnified scale. In drawing the curve, displacements of the particles towards the right of their mean positions are considered positive and those to the left negative. With this convention it will be seen from the figure that each condensation extends to the *right* of a crest on the displacement curve as far as the next trough. Points of maximum displacement are boundaries between neighbouring condensations and rarefactions, the condensation lying on that side of the particle towards which it is displaced. The density of the medium is normal at the points of maximum displacement. Both condensations and rarefactions have their maximum intensities at their centres, where there is an undisplaced particle. This is because the slope of displacement curve is greatest at these points, so that the distance between consecutive particles, which is equal to the *difference* of their displacements, is therefore either a maximum (rarefaction) or a minimum (condensation). Thus in a continuous medium in which a longitudinal wave is travelling, points of zero displacement are maxima or minima of density (and pressure) according as they are centres of condensation or rarefactions.

The particular type of progressive longitudinal wave which is known as **sinusoidal** is of exactly the same nature as a sinusoidal transverse wave except, of course, in respect of the direction of the displacements. It consists of a line of particles, all performing S.H.M. with the same frequency, the phase of the oscillations varying by equal amounts for equal increments of distance along the line. The wave depicted in Fig. 410 is actually sinusoidal. The equation representing the wave is exactly the same as for the transverse wave, that is

$$y = a \sin \frac{2\pi}{\lambda}(x - ct)$$

where y now signifies displacement from the mean position parallel to the direction of propagation, *i.e.* in the x direction.

The movements of the ear-drum which convey the sensation of sound are due to the rise and fall of air pressure in front of the drum caused by the arrival of condensations and rarefactions. These have been set up by the oscillations of the sounding body and have travelled through the air by the mechanism just described.

The Energy of a Progressive Wave.—When a particle performs S.H.M. of constant amplitude its total mechanical energy remains constant although the separate amounts of potential and kinetic energies are continually changing. The potential energy of the particle in any position is equal to the work which must be done against the restoring force in order to displace it from its equilibrium position to the point in question, while its kinetic energy is due to its mass and velocity. It follows that the particle has no potential energy when it is passing through its equilibrium position. At that instant its energy is wholly kinetic. We can therefore calculate the (constant) total energy of the particle by evaluating its kinetic energy as it passes through its mean position. Its velocity (known as the **particle velocity**) at this point is $a\omega$ (page 37, Vol. 1), where a is the amplitude of the oscillation and ω is equal to $2\pi/T$ or $2\pi n$, T being the time period and n the frequency of the oscillation. Therefore the kinetic energy of a particle of mass m is equal to

$$\tfrac{1}{2}ma^2\omega^2$$

or

$$2\pi^2 ma^2n^2$$

When a sound wave is being propagated in a medium, the total energy in any given volume is the sum of all the above terms for each particle in the volume. Thus for unit volume of a medium of density ρ we have

$$\begin{aligned} \text{energy per unit volume (or energy density)} &= \Sigma 2\pi^2 ma^2n^2 \\ &= 2\pi^2 a^2n^2 \Sigma m \\ &= 2\pi^2 \rho a^2 n^2 \end{aligned}$$

since Σm is equal to ρ, the mass of unit volume. Therefore for a sound of given frequency the energy is proportional to the square of the amplitude of the wave, and for a given amplitude it is proportional to the square of the frequency.

The Intensity of Sound.—The inten... .. a sound at any place is defined as the rate of flow of energy across unit area perpendicular to the direction of propagation. This is known as the **energy flux.** Since the speed of propagation is c, a wave train of length c crosses any area perpendicular to the direction of propagation in 1 sec., and if the area is unity the volume containing the waves which cross the area in 1 sec. is also c. Thus the energy flux, *i.e.* the intensity of the sound, at the point considered is c multiplied by the energy density, *i.e.*

$$2\pi^2 \rho c a^2 n^2$$

If a small source of sound is emitting E ergs of energy per sec. uniformly in all directions and there is no absorption in the surrounding medium, then at a distance r from the source E ergs per sec. pass through a spherical surface of radius r. The intensity or energy flux at this distance is therefore $\frac{E}{\text{area of sphere}}$, *i.e.* $\frac{E}{4\pi r^2}$. Thus the intensity obeys an **inverse-square law** under these conditions.

3. STATIONARY WAVES

Stationary or standing waves are of great importance in acoustics and elsewhere. They are produced when two progressive waves which are exactly similar in all respects are propagated in *opposite* directions, through a medium. The progressive waves may be either both transverse or both longitudinal. We shall examine the properties of the resulting stationary wave both graphically and analytically.

Graphical Treatment.—In Fig. 411 the straight line AG represents the undisturbed position of a line of particles over which two similar but opposite sinusoidal waves are passing. The displacement curves of the two waves at various instants are indicated by the dotted and broken lines respectively. In order to find the resultant displacement at any point and time we add algebraically the separate displacements due to the two progressive waves. This **principle of superposition** is fully justified by the agreement between theory and experiment.

We have chosen to begin the examination of the effect at the instant when the two displacement curves are completely coincident with each other (Fig. 411 (i)). The resultant displacement at every point is therefore double the displacement due to either of the separate waves and it is indicated by the full line. The state of affairs at a slightly later time (actually one-eighth of a time period) is shown in (ii). The full line again represents the resultant curve. Each of the subsequent drawings shows

the conditions at intervals of one-eighth of a period and is self-explanatory.

The reason why the resultant wave is called stationary or standing is now apparent. The points A, C, E and G, which are called **nodes,** have zero displacement at all times. Similarly the points B, D and F,

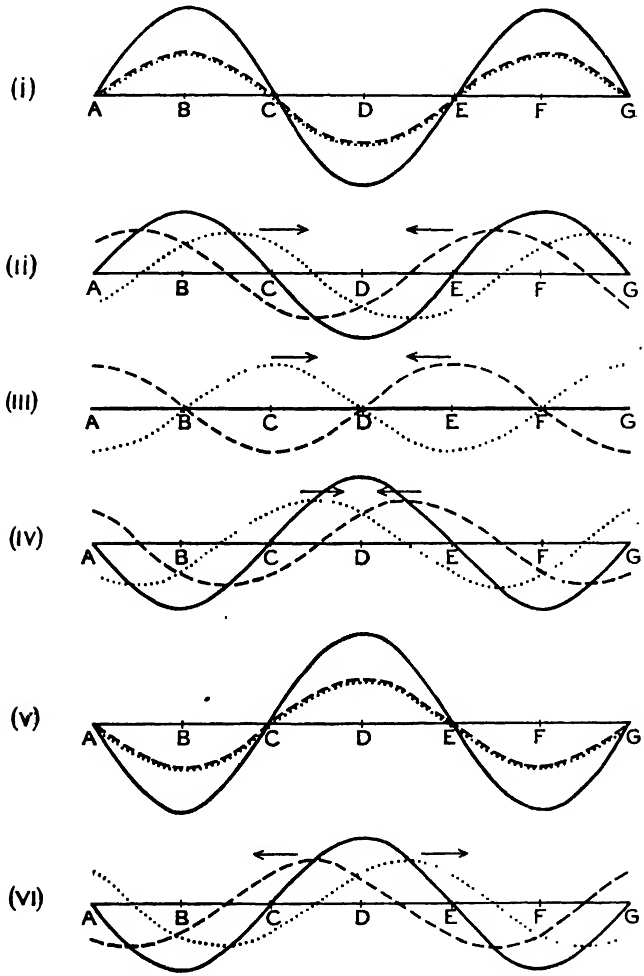


Fig. 411

known as **antinodes,** always have greater displacements than all other points, except at the instants when all displacements are zero. Thus the positions of nodes and antinodes remain constant while the displacement everywhere (except at the nodes) alters with time.

The motion of the particles can be described as follows. All particles

perform S.H.M. with a frequency which is the same as that of the component progressive waves. The particles between any two consecutive nodes all keep in phase with each other as they oscillate. This is because in respect of one of the progressive waves there is a phase lag between any two given particles, and in respect of the other wave there is an equal phase lead. The phases of the vibrations on the two sides of each node differ by π, which means that they are opposite to each other. The amplitude of the vibrations of the particles varies from zero at the nodes to a maximum at the antinodes, where it is equal to twice the amplitude

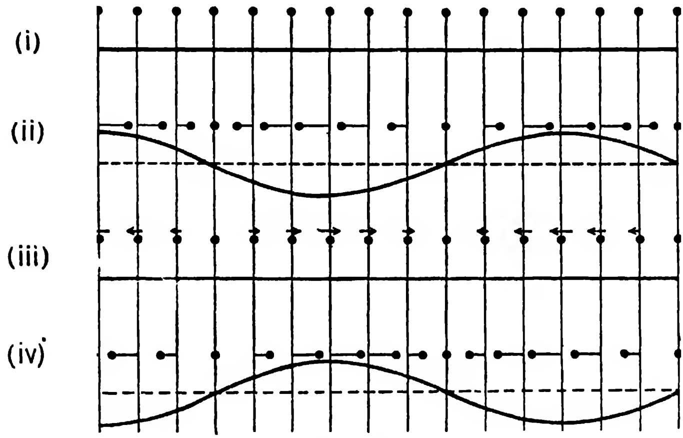


FIG. 412

of each of the component waves. It follows also that the particle velocity is always greatest at the antinodes and, of course, zero at the nodes. The wave-length of the stationary wave is the shortest distance between two points in the same phase, *e.g.* the distance between two *alternate* nodes or antinodes. This is the same as the wave-length of each of the progressive waves.

Fig. 412 shows the successive positions of a line of particles carrying a *longitudinal* stationary wave at intervals of one-quarter of a period, beginning in (i) at the stage when the particles are undisplaced. This stage corresponds to (iii) of Fig. 411. The thick line in each diagram is the displacement curve, and the arrows above the particles indicate their velocities. In (i) the pressure and density are uniform all along the line. From (i) to (ii) condensations and rarefactions develop, each extending from one antinode to the next, with nodes at their centres. In (iii) the pressure is once again uniform, and in (iv) a condensation has formed where there was previously a rarefaction and *vice versa.* Thus any given node is alternately the centre of a condensation and a rarefaction, the

intensities of which are continuously changing with time in a sinusoidal manner. An ear placed anywhere along the wave except exactly at an antinode would hear a musical sound of the same frequency as the sources of the progressive waves.

Analytical Treatment.—Suppose that the displacements due to the separate progressive waves are y_1 and y_2. Then bearing in mind that the waves have the same amplitude, frequency, wave-length and velocity but are travelling in opposite directions, we can write, as explained on page 559,

$$y_1 = a \sin \frac{2\pi}{\lambda}(x - ct)$$

$$= a \sin 2\pi\left(\frac{x}{\lambda} - nt\right)$$

and

$$y_2 = a \sin \frac{2\pi}{\lambda}(x + ct)$$

$$= a \sin 2\pi\left(\frac{x}{\lambda} + nt\right)$$

The first wave is travelling to the right, *i.e.* in the positive direction, while the second is travelling to the left. The resultant displacement at any time and place is given by the sum of the separate displacements, and remembering that for any pair of angles θ and ϕ,

$$\sin \theta + \sin \phi = 2 \sin \frac{\theta + \phi}{2} \cdot \cos \frac{\theta - \phi}{2}$$

we can write down the resultant displacement y at any point and time as follows:—

$$y = y_1 + y_2 = a\left\{\sin 2\pi\left(\frac{x}{\lambda} - nt\right) + \sin 2\pi\left(\frac{x}{\lambda} + nt\right)\right\}$$

$$= 2a \sin \frac{2\pi x}{\lambda} \cdot \cos 2\pi nt$$

This expression for y shows that the displacement varies periodically both with distance and time. Thus at any particular point the value of x is constant and we can write

$$y = A \cos 2\pi nt \quad . \quad . \quad . \quad . \quad (2)$$

where

$$A = 2a \sin \frac{2\pi x}{\lambda} \quad . \quad . \quad . \quad . \quad (3)$$

Thus according to equation (2) any given particle, specified by a given value of x (*i.e.* of A), performs S.H.M. with a frequency n and an amplitude A.

Every particle over which the two progressive waves are passing, therefore, performs S.H.M. (with certain exceptions, as we shall see immediately) with an amplitude which is constant for any given particle but which varies along the line of particles according to equation (3). For values of x which make A zero the amplitude is zero, which means that there is no vibration. This occurs at points where

$$2a \sin \frac{2\pi x}{\lambda} = 0$$

i.e. where

$$\frac{2\pi x}{\lambda} = 0, \pi, 2\pi, 3\pi, \text{etc.}$$

or

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \text{etc.}$$

These points are the stationary nodes, and the distance between consecutive nodes is $\lambda/2$. Furthermore, according to equation (3), the particles which have the greatest displacement at any time (regardless of algebraic sign, which determines whether the displacement is above or below the undisturbed line) are those particles for which x has the value necessary to give $\sin \frac{2\pi x}{\lambda}$ its maximum numerical value of unity. Thus the displacement is always greatest where

$$\sin \frac{2\pi x}{\lambda} = \pm 1$$

i.e. where

$$\frac{2\pi x}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \text{etc.}$$

or

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \text{etc.}$$

These points are the antinodes and are stationary points (since their positions are independent of time) situated midway between the nodes.

At each node the value of $\sin \frac{2\pi x}{\lambda}$ passes through zero and changes its algebraic sign, which means that when, at any time, the displacement is upwards on one side of a node it is downwards on the other side and *vice versa*, as illustrated in Fig. 411. This fact may be stated otherwise by saying that the phase of the oscillation differs by π on the two sides of each node.

EXAMPLES XXXIV

1. Draw a curve to represent the displacement of particles of air disturbed by sound waves from a tuning-fork.

Below this curve draw two others, showing the variation of pressure and of particle velocity along the waves, assuming them to be travelling from left to right across the paper.

What features of the curves determine the loudness and pitch of the sound heard by an observer? How would the displacement curve be altered if the sound was a violin or organ pipe? (L.I.)

2. Explain how a stationary undulation is produced by two equal waves travelling through a medium in opposite directions.

Describe how such undulations may be shown experimentally. (L.I.)

3. Express either graphically or by means of an equation a simple harmonic wave.

Show that two similar waves travelling in opposite directions combine to form a stationary undulation. (L.I.)

A wave-train represented by the equation

$$y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

is reflected normally without diminution of amplitude from a rigid wall. Find the equation representing the stationary wave motion produced, and show how it indicates the positions of the nodes and antinodes. How would you demonstrate the existence of these nodes and antinodes in a particular case? (L.I.)

5. A sharp sound is made near a row of equally spaced obstacles, *e.g.* wood posts. Explain why the reflected sound has a note of definite pitch.

Supposing the obstacles to be 9 in. apart, calculate the frequency of the note heard (*a*) by an observer near the posts and close to the source of sound, (*b*) by an observer at a considerable distance on a line perpendicular to the row. (The velocity of sound is 1110 ft. sec.^{-1}.) (L.I.)

6. Interpret the expression $y = a \sin \frac{2\pi}{\lambda}(vt - x)$ as applied to the propagation of sound waves. Discuss the nature and characteristics of the resulting motion due to the superposition of two identical wave trains moving in opposite directions. Give a short account of any acoustic measurements utilizing such a system.

A train of plane sound waves traverses a medium and the individual particles execute a periodic motion such that their displacement (in cm.) is given by $y = 5 \times 10^{-6} \sin (800\pi t + \theta)$.

(i) What is the amplitude of the motion?
(ii) Calculate the wave-length of the waves.
(iii) Calculate the phase difference in degrees between two particles 17 cm. apart at any *given instant*.

(Velocity of sound in medium $= 3{\cdot}4 \times 10^4$ cm. sec.^{-1}.) (L.Schol.)

Chapter XXXV

THE SPEED OF PROPAGATION OF SOUND WAVES

1. THEORY

*** Calculation of the Speed of Longitudinal Waves in a Fluid.**—Suppose that a longitudinal wave is travelling through a fluid medium with a speed c, and consider a column of the medium of unit cross-section with its axis parallel to the direction of the wave. This is represented in Fig. 413, where the vertical lines indicate the instantaneous positions of layers of particles which in the undisturbed medium would be equidistant from each other. At any instant the layers are in motion with different velocities parallel to the axis of the column, some moving to the right and some to the left—except, of course, those layers which, being instantaneously at the extremity of their oscillation, are at rest.

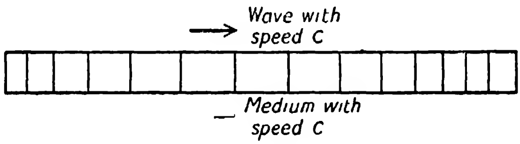


FIG. 413

Since c is the velocity with which the wave (*i.e.* any chosen displacement, condensation or rarefaction) travels with respect to the medium, it follows that if we suppose the medium to be travelling in the opposite direction to the wave with a velocity c, the effect would be to cause the series of condensations and rarefactions to become stationary with respect to an observer who is at rest. There would thus be a stream of fluid which, as it flows from right to left in Fig. 413, is alternately compressed and rarefied as it passes through certain fixed regions.

The velocity of the particles passing through any given position (*e.g.* the centre of a condensation) would be the algebraic sum of c and the instantaneous velocity which they possess by virtue of the wave motion. In the original progressive wave, the particles in a condensation are moving forward in the direction of propagation with various velocities according to their positions in the condensation, those in the centre moving most rapidly. Similarly the particles in a rarefaction are moving against the direction of propagation. The particle velocities are, however,

* The actual derivation of equation (5) is more advanced than the general scope of this book.

always less than c, so that when the velocity c is superimposed against the direction of the wave all the particles move in this backward direction, those in a rarefaction moving faster than c and those in a condensation more slowly. It will be noticed that this is in conformity with Bernoulli's equation (page 207, Vol. 1), according to which the velocity of a moving fluid is large where the pressure is low (rarefaction) and small where the pressure is high (condensation). The stream is speeded up as it travels from the centre of a condensation to the centre of the next rarefaction, after which it is retarded until the centre of the next condensation is reached and so on. An expression for the speed of the waves can be derived from Bernoulli's theorem or, as we shall do, by applying Newton's second law of motion directly.

In the stream of fluid created by superimposing the velocity c from right to left consider the flow between a plane A (Fig. 414) where the pressure p and density ρ are normal and the velocity of the particles is c, and another plane B a very short distance Δl from A. The plane A is the transition layer between a condensation and a rarefaction since the particle velocity is zero here. Let the pressure and density at B be $p+\Delta p$, $\rho+\Delta\rho$, respectively. Since the pressure is greater at B than at A the velocity must be less, and we can write it as $c-\Delta u$, where Δu is the speed which the particles at B would have by virtue of the wave motion alone. This will be a very small speed, of the same order of magnitude as Δp and $\Delta\rho$, if Δl is small, because at A the particle speed due to the wave itself is zero.

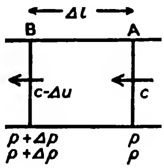


FIG. 414

The volume of fluid crossing A per second is c (since the column has unit cross-section), and the mass of this fluid is $c\rho$. Similarly, the mass crossing B is $(c-\Delta u)(\rho+\Delta\rho)$. There is no change in the mass of fluid between A and B as time goes on, and therefore the masses entering at A and leaving at B are equal, and we have

$$(c-\Delta u)(\rho+\Delta\rho)=c\rho$$

or

$$c\Delta\rho=\rho\Delta u \quad . \quad . \quad . \quad . \quad . \quad (1)$$

since we can neglect the product of the two small quantities $\Delta u . \Delta\rho$.

The time for the particles to pass from A to B can be written $\frac{\Delta l}{c}$, only a negligible error being made in taking the average velocity between A and B to be c instead of slightly less than c, as it actually is, since Δl, and therefore Δu, can be made as small as we please. During this time the speed of the particles decreases by Δu, so that they suffer an acceleration of $\Delta u \div \frac{\Delta l}{c}$ directed towards the right. Again, the mass of fluid between A and B is $\rho\Delta l$ (with a negligible error due to the fact that the average

density is really slightly greater than ρ), so that there is a retarding force acting on it equal to the product of its mass and its acceleration, *i.e.* to

$$(\rho \Delta l . \Delta u) \div \frac{\Delta l}{c}$$

or

$$\rho c \Delta u$$

This force is equal to Δp, so that

$$\Delta p = \rho c \Delta u$$

or, by equation (1),

$$\Delta p = c^2 \Delta \rho \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the bulk modulus of elasticity of the fluid (page 217, Vol. 1) is k we ha e, by definition,

$$k = \Delta p \div \frac{\Delta V}{V} \quad . \quad . \quad . \quad . \quad (3)$$

where V is the volume of a quantity of the fluid under a pressure p, and ΔV is the decrease of this volume when the pressure rises by Δp. Considering a mass m we have

$$V = \frac{m}{\rho}$$

and

$$V - \Delta V = \frac{m}{\rho + \Delta \rho}$$

Therefore, by subtraction we have

$$\Delta V = \frac{m}{\rho} - \frac{m}{\rho + \Delta \rho}$$

$$= \frac{m \Delta \rho}{\rho(\rho + \Delta \rho)}$$

$$= \frac{m \Delta \rho}{\rho^2} \quad \text{if } \Delta \rho \text{ is very small}$$

$$= \frac{V \Delta \rho}{\rho} \quad \text{since } V = \frac{m}{}$$

$$\therefore \frac{\Delta V}{V} = \frac{\Delta \rho}{\rho}$$

Substituting in equation (3) we obtain

$$k = \frac{V \Delta p}{\Delta V}$$

$$= \frac{\rho \Delta p}{\Delta \rho} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Finally therefore we obtain, from equations (2) and (4),

$$c^2=\frac{\Delta p}{\Delta \rho}$$

$$=\frac{k}{\rho}$$

or

$$c=\sqrt{\frac{k}{\rho}} \qquad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

This equation is a particular form of a more general equation, namely

$$c=\sqrt{\frac{\text{Elasticity modulus}}{\text{Density}}}$$

which is applicable to the various forms of wave motion in solids and fluids provided that the elasticity modulus inserted in the equation is the appropriate modulus for the particular type of deformation involved in the wave propagation. Thus, as we have seen, the bulk modulus is appropriate when longitudinal waves pass through a fluid because the medium is alternately compressed and rarefied. When a longitudinal wave is transmitted along a solid rod the material is alternately stretched and compressed parallel to the axis of the rod, and the elasticity referred to in the equation is Young's modulus. The fact that the speed of waves depends on the ratio of elasticity to density was anticipated by a qualitative argument on page 554.

The Speed of Sound in a Gas.—It is shown on page 227 (Vol. 1) that when a gas which obeys Boyle's law is compressed or expanded by changing its pressure at constant temperature (*i.e.* isothermally) its bulk modulus is equal to its pressure. Newton used a value of k equal to atmospheric pressure in order to calculate the speed of sound in air from equation (5). For air at S.T.P. (remembering that pressure must be expressed in dynes cm.$^{-2}$ and that standard atmospheric pressure is about 10^6 in these units) we have

$$c=\sqrt{\frac{10^6}{0{\cdot}00129}}$$

$$=280\times10^2 \text{ cm. sec.}^{-1} \quad \text{approximately}$$

This value is much lower than the observed value of about 330 m. sec.$^{-1}$, and for many years the discrepancy was not properly accounted for, although Newton himself put forward a rather facile explanation in terms of the finite sizes of air particles.

Eventually Laplace solved the problem in the early nineteenth century. He argued that it was unlikely that the heat generated during the formation of a condensation would be dissipated immediately, and that therefore the

temperature of any given portion of the gas would rise as a condensation passed through it and fall during the subsequent expansion when it was rarefied. The changes of volume would therefore not be isothermal but would be adiabatic, *i.e.* of the kind which occurs when heat is prevented from entering or leaving a given quantity of gas when its volume is changed. The bulk modulus of an ideal gas under adiabatic conditions is shown on page 228 (Vol. 1) to be γp instead of p, where γ is the ratio of the specific heats at constant pressure and constant volume. Laplace's expression is, therefore,

$$c=\sqrt{\frac{\gamma p}{\rho}} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

which gives about 330 m. sec.$^{-1}$ for air when the appropriate value of γ (about 1·41) is used. This value of c is therefore in remarkable agreement with observation, and it is now accepted that the adiabatic and not the isothermal bulk modulus is involved in the propagation of longitudinal waves through fluids. Indeed one of the most accurate methods of determining γ for a gas is by calculation from observations of the speed of sound through it.

The Influence of Temperature on the Speed of Sound in a Gas.—The pressure, volume and temperature of m gm. of a gas which can be regarded as **ideal** are (page 364, Vol. 2) related by the equation

$$pV=mRT$$

where R is the gas constant for unit mass of the particular gas and T is its absolute temperature, which is approximately equal to Centigrade temperature *plus* 273. Since the density is equal to m/V the equation can be written

$$\frac{p}{\rho}=RT \quad . \quad . \quad . \quad (7)$$

and combining this with equation (6) we obtain

$$c=\sqrt{\gamma RT}$$

Therefore for a given gas, since γ and R are constants, we have

$$c \propto \sqrt{T}$$

within the range in which the gas obeys the ideal-gas equation.

Suppose that the speed of sound in a given gas is c_0 at 0° C. and c_t at t° C. We then have

$$\frac{c_t}{c_0}=\frac{\sqrt{\gamma R(t+273)}}{\sqrt{\gamma R\times 273}}=\sqrt{\frac{t+273}{273}}$$

This is an exact equation which can be approximated to a more convenient form as follows. It can be written

$$c_t = c_0\left(\frac{t+273}{273}\right)^{\frac{1}{2}}$$

$$= c_0\left(1+\frac{t}{273}\right)^{\frac{1}{2}}$$

The right-hand side may be expanded by the binomial theorem and, if t is only a few degrees, $\frac{t}{273}$ is a small quantity of which the square and higher powers can be neglected by comparison with unity. Thus the approximate equation is

$$\left(1+\frac{t}{2\times 273}\right.$$

$$= c_0 + \frac{c_0 t}{2\times 273}$$

For air c_0 is about 330 m. sec.$^{-1}$, so that

$$\frac{c_0}{2\times 273} = \frac{330}{2\times 273}$$

$$= 0{\cdot}61 \quad \text{approximately}$$

A working formula for the speed of sound in air at a temperature of t° C. is therefore

$$c_t = (330 + 0{\cdot}61t) \text{ m. sec.}^{-1}$$

and is valid for comparatively small values of t.

The Influence of Pressure.—When the temperature of an ideal gas is constant its density is proportional to its pressure. This is a statement of Boyle's law and is implicit in equation (7). Since the speed of sound is equal to $\sqrt{\frac{\gamma p}{\rho}}$, and both γ and $\frac{p}{\rho}$ are constant for a given gas at a given temperature, it follows that the speed of sound is independent of pressure.

2. EXPERIMENTAL DETERMINATION OF THE SPEED OF SOUND

The Speed of Sound in Air.—Everyone has noticed the effects of the finite speed of propagation of sound. The sound of a cricket-ball being struck reaches an observer 100 yards or so away slightly after the actual event is seen to occur. Echoes take a finite time to travel to the reflecting surface and back again, and the lag between seeing a lightning flash and hearing the thunder is often very considerable.

Measurements of the speed of sound were conducted in the open air in the early seventeenth century, the source of sound often being a gun. An observer at a measured distance from the gun recorded the lapse of time between seeing the flash and hearing the report. The time taken for the light to travel is, of course, infinitesimal. Sound takes about 4·8 sec. to travel a mile, so that the time interval is always fairly short. This makes the determination very susceptible to the personal error of the observer, which is the inevitable lag between receiving the sensation (light or sound) and taking the necessary action such as starting and stopping a watch. In the nineteenth century Regnault attempted to eliminate the human element by recording the firing of the gun electrically and receiving the sound on a distant membrane which closed an electrical circuit when it was disturbed. The receiver was, however, necessarily rather insensitive so that only a comparatively short distance (2850 metres) could be used, and moreover there was still a lag during which the membrane moved from its undisturbed position to that in which it closed the circuit. During the war of 1914–18 a **hot-wire microphone** detector was devised for use in the location of enemy gun positions from observations of the arrival of their sound at various stations **(sound ranging).** This instrument consists of a fine resistance wire placed in the mouth of a large air cavity. The wire forms one arm of a balanced Wheatstone bridge circuit, and a steady electric current is passed through it. When sound waves reach this arrangement the condensations and rarefactions cause air to flow into and out of the cavity past the wire, which is consequently cooled and its resistance changes. The galvanometer of the bridge circuit therefore shows a deflection due to the disturbance of the balance when the sound arrives. This arrangement constitutes a very sensitive sound detector and has been used in determinations of the speed of sound in the open air.

Open-air experiments conducted over long distances are subject to the effects of wind, since sound travels with a definite velocity c relative to the air irrespective of the motion of the air itself. The velocity of sound actually observed is therefore the vector sum of two velocities. Reciprocal observations in opposite directions are always made in order to eliminate this effect, but wind speed may not be uniform over the whole distance and may vary with time. Temperature and humidity also differ from place to place so that outdoor experiments can never be made under controlled conditions.

In 1905 Hebb in America carried out an indoor determination of the speed of sound in air which amounted essentially to a determination of the wave-length of a sound wave of known frequency. The arrangement of his apparatus is shown in Fig. 415. Two large paraboloidal reflecting surfaces M_1 and M_2 made of plaster of paris were set up facing each other, and telephone transmitters T_1 and T_2 were placed at their foci. A whistle W, blown at a steady pressure, was placed near T_1. The secondary coil S

of a special transformer having two separate primaries P_1 and P_2 was joined to a telephone receiver R, while P_1 and P_2 were separately connected to T_1 and T_2 respectively, a battery being included in each of these circuits. Sounds reaching either of the telephone transmitters thus caused fluctuating electric currents in the corresponding primary coil which in turn gave rise to induced currents in S. In this way a sound received by either T_1 or T_2 was reproduced in R, and if both transmitters were receiving sounds simultaneously the sound in R was their resultant. Now T_1 received the sound direct from W almost instantaneously, while T_2 picked up the waves which had travelled by the path involving two reflections which is shown by dotted lines in the drawing. If the difference between the lengths of the paths from W to T_1 and from W to T_2 was a whole number of wave-lengths, then at the instant when a condensation arrived at T_1 another condensation (which had been emitted by W earlier) arrived at T_2. Similarly rarefactions arrived simultaneously. Thus the sounds reinforced each other and there was a maximum sound intensity in R. The direct opposite of this occurred if the paths travelled by the two waves differed by an odd number of half wave-lengths, because then a condensation arrived at T_2 when T_1 was receiving a rarefaction and *vice versa*, so that the two sounds in R cancelled each other and there was silence. The experiment consisted in gradually moving M_2 and T_2 away from M_1 and noting the distances between consecutive positions for which there was silence in R. The mean of these distances was one wave-length, and could be accurately determined because about 200 positions of silence were observed over a total distance of about 100 ft. The speed of sound was then obtained by multiplying the wave-length by the frequency of the whistle, which was determined by comparison with a standard tuning-fork.

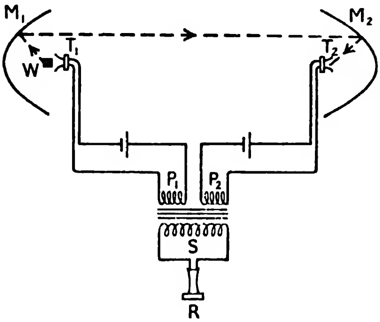


FIG. 415

Within the range of frequencies to which the ear is sensitive there is apparently no measurable variation of the speed of sound with frequency. If such an effect existed, the music from a band would be distorted after the sound had travelled a considerable distance, but this is not observed to occur.

In some of the experiments on the propagation of sound from large explosions it appears that the speed of sound is abnormally high very near the source, but it is rather doubtful whether we can infer that this is due solely to the large intensity.

The Speed of Sound in Water.—The earliest determination of the speed of sound in water was made on Lake Geneva in 1827 by means of a bell hung beneath a boat. When the submerged bell was struck a simultaneous flash was made, and an observer in another boat about 14 km. away noted the time between seeing the flash and receiving the sound in a submerged horn connected to a listening tube. A value of 1440 m. sec.$^{-1}$ was obtained.

Since then more elaborate and accurate determinations have been made, particularly after the First World War and in connection with the development of **echo depth sounding.** This device enables a vessel to estimate the depth of the sea by observing the time taken for a sound to travel down to the bottom—where it is reflected—and back again. An accurate knowledge of the speed of sound in sea-water is required in the calculation of the depth, and several independent determinations have been made by recording electrically the arrival of the sound of an underwater explosion at each of a series of submerged microphones (called "hydrophones") whose positions are accurately known.

3. THE DOPPLER EFFECT

One of the commonest examples of this effect occurs when a railway locomotive sounding its whistle passes an observer who is stationary or travelling in the opposite direction in another train. After passing the observer, the whistle has a noticeably lower pitch than it had before. The effect is also observable with the horn of a fairly swiftly moving car. We shall proceed to show theoretically that the wave-length of the sound waves emitted by a given source is, in general, modified by the motion of the source through the air, and also that the apparent frequency of sound waves depends upon the motion of the observer relative to them. The effects of the motion of the source and of the observer are quite separate and should not be confused. We consider the former first.

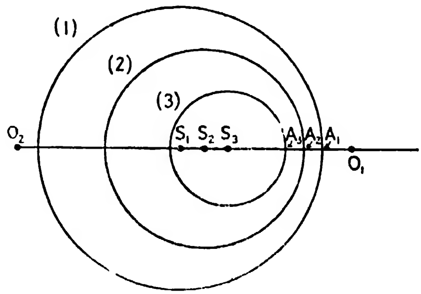


FIG. 416

Effect of Motion of the Source.—Suppose that a source of sound of frequency n is moving through still air with a velocity u. When the source was at S_1 (Fig. 416) it gave out a compression which travelled with the same velocity c in all directions and, at the instant considered in the figure, has reached the circumference of circle (1), the centre of which is S_1. The next compression was given out $1/n$ sec. later when the source was

at S_2, and it has travelled to the circumference of circle (2) which has S_2 as its centre. Similarly S_3 is the centre of circle (3), which represents the compression given out by the source when it was at S_3 $1/n$ sec. after passing through S_2. The difference between the radii of successive circles is λ, the wave-length of the waves which the source emits when it is stationary. This is equal to c/n. But owing to the movement of the source the circles are not concentric, and to an observer at a point such as O_1 on the path of the source the wave-length is less than λ. Let it be λ'. Then

$$\begin{aligned}\lambda' &= A_1A_2 = A_2A_3, \text{ etc.}\\ &= S_1A_1 - S_1A_2\\ &= S_1A_1 - S_2A_2 - S_1S_2\\ &= \lambda - S_1S_2\\ &= \lambda - \frac{u}{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)\end{aligned}$$

since the source travels the distance S_1S_2 in $1/n$ sec. with a speed u.

Since compressions separated by a distance λ' are travelling ahead of the source with a speed c, the number, n', arriving per second at any point such as O_1 will be equal to c/λ'. This, therefore, is the apparent frequency of the source to an observer at O_1. Equation (8) can therefore be written

$$n' \quad n \quad n$$

or

$$nc = n'c - n'u$$

so that

$$n' = n \cdot \frac{c}{c-u} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

By similar reasoning the apparent frequency to an observer situated at a point such as O_2 in the path of the source but behind it is shown to be equal to

$$n \cdot \frac{c}{c+u} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

We can therefore state that equation (9) applies to each case of motion of the source of sound along the straight line joining it to the observer provided that u is regarded as positive when the source is moving towards the observer and negative when it is moving away. We could, of course, equally well adopt the other formula (10) together with the reverse convention with regard to the sign of u. When the source passes the observer, its apparent frequency changes from the value given by (9) to that given by (10), *i.e.* from a frequency which is higher than its actual frequency to one which is lower. For example, if a locomotive whistle approaching

an observer at 50 m.p.h. has an actual frequency of 1000 cycles per sec., its apparent frequency is

$$1000 \times \frac{750}{750-50}$$

$$= 1071 \text{ cycles per sec. (c.p.s.) approximately}$$

if we take the speed of sound in air to be 750 m.p.h. Similarly when it is receding with the same speed the whistle has an apparent frequency of

$$1000 \times \frac{750}{750+50}$$

$$= 938 \text{ c.p.s. approximately}$$

Reference to Fig. 416 will show that the greatest difference between the real and apparent frequencies occurs at points lying on the straight line along which the source is moving at any instant. The effect is smaller at other points, and zero at points on the line passing through the source and perpendicular to the path.

Effect of Motion of the Observer.—We now consider the case of a moving observer. Suppose that the observer at O_1 in Fig. 416 moves *towards* the source with a velocity v. The observer's velocity relative to the compressions is $c+v$. Consequently the number of compressions passing him per second is the number of compressions contained in a length $c+v$ of the wave train, which, by proportion, is

$$n' \cdot \frac{c+v}{c} \qquad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

since, as we have just seen, there are n' compressions in a distance c. This expression is therefore the frequency which the source appears to have to an observer moving towards it with a velocity v. If the observer is moving *away from* the source with a velocity v, his velocity relative to the waves is $c-v$ and the apparent frequency is

$$n' \cdot \frac{c-v}{c} \qquad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Thus we can use formula (11) for both cases provided that v is regarded as positive or negative according as it is directed towards or away from the source.

Substituting expression (9) for n' in formula (11) we obtain the general expression for the apparent frequency in terms of the actual frequency as follows:—

$$\text{Apparent frequency} = n \cdot \frac{c+v}{c-u}$$

together with the convention as to the signs u and v already stated.

When both the source and the observer are moving in the same direction with the same speed, u and v are equal in magnitude but opposite in sign, and the factor $\frac{c+v}{c-u}$ is unity. Thus the Doppler effect is absent when there is no relative motion between the source and the observer. It should be noticed, however, that the relative velocity does not enter into the formula, and that for the same relative velocity the magnitude of the Doppler effect depends upon the actual velocities of the source and observer with respect to the medium.

In general, when the motions of the source and observer are not directed along the line joining them, u and v must be regarded respectively as the instantaneous components along the line joining them of the respective velocities of the source and observer.

The effect of a wind is to modify the velocity of the sound waves. To take a simple case, if the wind is blowing directly from the source to the observer in Fig. 416 with a speed w, it is necessary to add w to c in the formula. For given values of u and v, therefore, the magnitude of the Doppler effect is in general dependent upon w. When u and v are both zero, however, or when $u = -v$ (no relative motion between source and observer), there is no difference between the real and apparent frequencies whatever may be the value of w.

EXAMPLES XXXV

1. A ship at sea sends out simultaneously a wireless signal above the water and a sound signal through the water, the temperature of the water being 4° C. These signals are received by two stations, A and B, 25 miles apart, the intervals between the arrival of the two signals being $16\frac{1}{2}$ sec. at A and 22 sec. at B. Find the bearing from A of the ship relative to AB. The velocity of sound in water at ° C. $=4756+11t$ ft. per sec. (J.M.B.H.S., abridged.)

2. Explain the Laplace correction to Newton's formula for the velocity of sound in a gas.

Calculate the velocity of sound at 0° C. in a gas for which the specific heat at constant pressure is 0·235 and the specific heat at constant volume 0·175. (The mechanical equivalent of heat is 4·2 joules per calorie.) (L.I.)

3. At what temperature is the velocity of sound in a gas 10 per cent. higher than at 0° C.? (L.I., abridged.)

4. Explain Laplace's modification of Newton's formula for the velocity of sound in a gas.

It is commonly stated that the velocity of sound in air increases by 61 cm. sec.$^{-1}$ for each Centigrade degree rise of temperature. How is this figure obtained? In what circumstances may it be used? If it is correct for a rise of temperature from 0° C. to 1° C., what is the velocity at 0° C.? (L.I.)

5. Describe a determination (other than by resonance) of the velocity of sound in air. How is the velocity dependent upon atmospheric conditions?

Give Newton's expression for the velocity of sound in a gas, and Laplace's correction. Hence calculate the velocity of sound in air at 27° C. (Density of air at S.T.P. $=0·00129$ gm. cm.$^{-3}$; $C_p=0·24$ cal. gm.$^{-1}$ deg.$^{-1}$ C.; $C_v=0·17$ cal. gm.$^{-1}$ deg.$^{-1}$ C.) (L.H.S.)

6. State briefly how you would show by experiment that the characteristics of the transmission of sound are such that (*a*) a finite time is necessary for transmission, (*b*) a material medium is necessary for propagation, (*c*) the disturbance may be reflected and refracted.

The wave-length of the note emitted by a tuning-fork, frequency 512 vibrations per sec., in air at 17° C. is 66·5 cm. If the density of air at S.T.P. is 1·293 gm. per litre, calculate the ratio of the two principal specific heats of air. Assume that the density of mercury is 13·6 gm. per c.c. (J.M.B.H.S.)

7. Describe the way in which a longitudinal progressive wave travels, explaining how the motions executed by individual particles of the medium are related to the propagation of the wave itself.

Discuss the conditions under which a stationary wave system can be set up, and describe an experimental arrangement for realizing those conditions.

With a source of sound of frequency 5000 cycles per sec., stationary waves are formed in air at 0° C. The distance between successive antinodes is 3·31 cm. At a higher temperature it is found that the separation of the antinodes, with the same source, is 3·43 cm. Calculate (*a*) the velocity of sound in air at 0° C., (*b*) the temperature at which the second observation was made. (O.H.S.)

8. Give an expression for the speed of sound in a gas, explaining the meanings of the symbols you employ. How does the speed depend upon the temperature and the pressure of the gas?

The speed of sound in dry air at S.T.P. is 331 metres sec.$^{-1}$. Assuming the volume composition of air to be 4/5ths nitrogen (atomic weight 14) and 1/5th oxygen (atomic weight 16), calculate the speed of sound in nitrogen under the same conditions. At what temperature would the speed in oxygen be the same as that in nitrogen at 0° C.? (L.Med.)

9. On a day when the temperature of the atmosphere at ground-level is 15° C., it is found that the temperature decreases by 6° C. for each kilometre rise. If the velocity of sound is 31,500 cm. sec.$^{-1}$ at sea-level, what is its value at a height of 10 kilometres? (O.H.S., abridged.)

10. Explain how the pitch of a note is affected by the motion of the source and of the observer.

An express train blowing its whistle passes through a station, and an observer standing on the platform notices that, as the engine passes him, there is a drop in pitch corresponding to a frequency change in the ratio 6 to 5. Calculate the speed of the train in miles per hour. (Velocity of sound in air = 1100 ft./sec.) (L.Med.)

11. Distinguish between the effects which modify the frequency of the note heard when (*a*) an observer, and (*b*) a source of sound are in motion.

An engine moving with a velocity of 60 miles per hour sounds a whistle of frequency 200 cycles per sec. What are the maximum and minimum frequencies heard, and where should an observer be placed in order to hear them? (Velocity of sound in air is 1100 ft. per sec.) (L.Med.)

12. A whistle is blown and at the same time moved towards a reflecting wall. State and explain the effects heard by an observer (*a*) stationary between the whistle and the wall, (*b*) moving with the whistle and (*c*) stationary behind the whistle, *i.e.* further from the wall than the whistle. (L.Med.Schol.)

13. A motor-car, travelling initially at 25 m.p.h., is approaching a stationary observer and emitting a sound whose frequency is proportional to the speed of the car. If the pitch of the note heard by the observer as the car recedes is the same as that heard initially, to what speed must the car have accelerated while passing? (Velocity of sound in air = 750 miles/hour.) (L.Med., abridged.)

14. Explain why the motion of a source of sound affects its pitch as heard by a stationary observer. How can the phenomenon be demonstrated in a classroom?

What is the velocity of the source along the line joining the source to the observer if, as a result of the motion, the frequency of the note heard is (*a*) increased in the ratio 16 : 15, (*b*) decreased in the ratio 15 : 16? Assume that the velocity of sound is 1120 ft. per sec. and give the results in feet per sec. Derive any formula employed. (J.M.B.H.S.)

15. Explain why the frequency of a wave motion appears, to a stationary observer, to change as the component of the velocity of the source along the line joining source and observer changes. Describe two illustrations of this effect, one with sound and one with light.

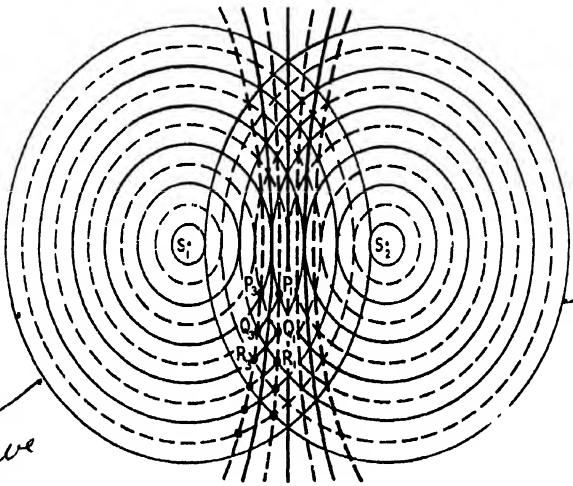
A stationary observer is standing at a distance l from a straight railway track and a train passes with uniform velocity v sounding a whistle with frequency n_0. Taking the velocity of sound as V, derive a formula giving the observed frequency n as a function of time. At which position of the train will $n = n_0$? Give a physical interpretation of the result. (C.H.S.)

Chapter XXXVI

THE BEHAVIOUR OF WAVES

1. INTERFERENCE

The Superposition of Two Spherical Waves.—In dealing with the properties of stationary waves (page 563) and their formation by two progressive waves we saw that, whereas a single progressive wave causes every particle over which it passes to vibrate with the same amplitude, the combination of the two waves gives rise to vibrations whose amplitude varies from zero at the nodes to a maximum at the antinodes. This is a particular case of "interference" between two wave trains.



second wave train

first wave train

FIG. 417

Consider two identical progressive waves emitted by two fixed point sources S_1 and S_2 (Fig. 417). It is simpler at first to consider the disturbance in one plane only, *i.e.* to confine our attention to circular waves, although in general the waves are spherical and travel in all directions from S_1 and S_2. Let the full circles represent the positions of the wave crests at a given instant, and let the intermediate broken circles represent the troughs. If we suppose that the amplitude is the same everywhere (which cannot be exactly true), we can say that at each of the points of intersection of a dotted circle with a full circle there is no disturbance of the medium. This is the case not only at the instant considered in the figure but at all times, because at such points the phases

of the two waves are *always* opposite, just as they are at the nodes of a stationary wave. In order that there shall be no disturbance at any particular point it is evidently necessary, therefore, that the distances of the point from S_1 and S_2 shall differ by an odd number of half wave-lengths. At a point where this condition is satisfied the displacement due to one of the waves is always equal and opposite to the displacement due to the other.

By inspection of Fig. 417 it will be seen that the so-called "path difference" is one half wave-length at points like P_1, Q_1, R_1, and it is three half wave-lengths at P_3, Q_3, R_3 and so on. When smooth curves are drawn through points having the same path difference, intermediate points on these curves also have the same path difference. This is made evident by imagining the trough and crest which coincide at, say, P_1 to move outwards from their respective sources. Their point of intersection then travels from P_1 to Q_1 along the curve joining these points. Thus at every point on any of the heavy broken curves there is no displacement at any time. It can be proved that these curves are hyperbolic in shape.

The intermediate full-line curves joining points of coincidence of trough with trough and crest with crest are the loci of points at which the vibrations due to each wave are exactly in phase, the path difference being a whole number of wave-lengths at every instant. Waves whose amplitude is the sum of the two separate amplitudes travel along these lines, which means that at every point on one of these lines the medium is vibrating with double the amplitude of the two separate waves. The energy of the disturbed medium is not uniformly distributed as it is when there is only one progressive wave. It varies from zero along the curves on which there is no vibration to a maximum along the curves of maximum vibration.

An interference pattern of the type just described can be produced on the surface of water or mercury by causing two thin rods touching the surface to vibrate with equal frequencies.

In sound, conditions similar to those depicted in Fig. 417 can be achieved by using two identical sources. The experiment is more effective if the frequency of each source is fairly high, so that the wave-length is short and the positions of maximum and minimum sound intensity are correspondingly closer together. If a single source of sound, such as a loud-speaker, emits a note of constant frequency in a room, many regions of maximum and minimum intensity are often observable. These are formed by interference between the sound waves reflected to the ear by the walls and those arriving directly. demonstration

A commonly-used device for demonstrating the interference of sound waves and measuring their wave-length is the **Quincke tube**, which is shown in Fig. 418. Sound waves from a high-frequency source S, such as a maintained whistle, enter the apparatus and divide, some of the energy travelling round A and the remainder round B. The two waves

reunite at C and are picked up by a detector D, such as a microphone or a **sensitive flame.** This latter device is a tall, steady high-pressure gas flame which ducks and roars in the presence of sound waves. The length of the path of the waves through B can be altered by moving the U-shaped tube in and out as in a trombone. Suppose that B is initially in such a position as to make the paths through A and B equal. The two waves will then arrive at D in phase and will produce the maximum sound or disturbance of the flame. When B is gradually withdrawn the intensity at D diminishes, and practically reaches zero when the path through B exceeds that through A by half a wave-length, because a condensation from one side arrives simultaneously with a rarefaction from the other. Further withdrawal of B brings another maximum followed by another minimum and so on. Evidently the increase in the length of the path through B between two consecutive maxima or minima is a wave-length, and the wave-length can therefore be determined.

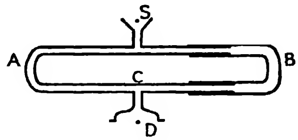
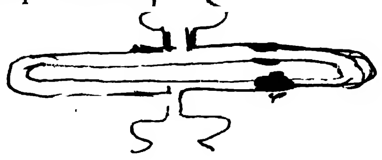


FIG. 418

Beats.—We have seen that interference between the waves from two sources of equal frequency gives rise to regions of maximum and minimum displacement. These regions are stationary with respect to the sources. We now discuss what happens if the two frequencies are not equal. It is evidently possible to find a point at which, say, wave crests are arriving simultaneously from each source at a certain instant. At such a point the disturbance is instantaneously a maximum, and we consider what happens there as time goes on. If the source S_1 has a higher frequency than S_2, the next crest from S_1 will arrive at the point considered slightly before the corresponding crest from S_2, so that the displacement at the end of the first oscillation will be less than the sum of the heights of the two crests. When the next crest arrives from S_1 the displacement due to S_2 will be even further from its maximum. Thus at the point we are considering, the phase difference between the vibrations due to the two separate waves becomes continuously greater, and the resultant vibration diminishes until a time is reached at which the phases are opposite. Provided that the amplitudes are equal there will then be instantaneously no resultant vibration, while if they are unequal there will be a vibration of minimum amplitude. After this the waves begin to get into step, and when S_1 has gained one complete vibration on S_2 they again reinforce each other and conditions are exactly as they were at the beginning of the interval. If S_1 and S_2 perform n_1 and n_2 oscillations per second respectively, then in one second the number of vibrations which S_1 gains on S_2 is $(n_1 - n_2)$. Thus S_1 gains a complete vibration on S_2 $(n_1 - n_2)$ times per second, and the amplitude at any given point becomes a maximum $(n_1 - n_2)$ times per second.



A simple analytical treatment of the phenomenon may be presented as follows. Suppose that we choose a point at which the amplitude of both waves is the same and equal to a. The displacement y_1 due to S_1 at any time can be written

$$y_1 = a \sin 2\pi n_1 t$$

supposing that the oscillations are simple harmonic. Similarly for the displacement y_2 due to S_2 we write

$$y_2 = a \sin 2\pi n_2 t$$

The form of the equations indicates that t represents the time which has elapsed since the instant at which both oscillations simultaneously passed through their zero at the point considered. Such instants occur periodically at every point so that we are justified in choosing one of them as our time zero. The resultant displacement y is given by

$$y = y_1 + y_2 = a \sin 2\pi n_1 t + a \sin 2\pi n_2 t$$

$$= 2a \sin 2\pi \frac{n_1 + n_2}{2} t . \cos 2\pi \frac{n_1 - n_2}{2} t \qquad (1)$$

This equation shows that there is an oscillation, represented by the sine

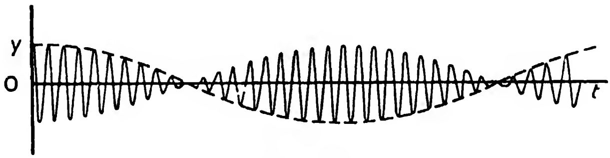


FIG. 419

term, of a frequency equal to the arithmetic mean of the two separate frequencies.

The amplitude of this oscillation, *i.e.* $2a \cos 2\pi \frac{n_1 - n_2}{2} t$, varies with time in a periodic manner with a frequency of $\frac{n_1 - n_2}{2}$. This quantity has a maximum *positive* value $\frac{n_1 - n_2}{2}$ times per second and an equal maximum *negative* value the same number of times. Thus the maximum disturbance from the zero position occurs $(n_1 - n_2)$ times per second. This is illustrated by Fig. 419, in which the full line shows the variation of y with t according to equation (1). The broken line is a graph of $2a \cos 2\pi \frac{n_1 - n_2}{2} t$ against t, and it indicates the magnitude of the maximum displacement of each oscillation both upwards and downwards.

The recurrence of maximum and minimum oscillations at every point where two waves of different frequency interfere is known as "beats." Beats between two sources of sound can be demonstrated very easily by

placing the ear or some artificial sound-detecting device at a spot where it receives with approximately equal intensity the waves from two similar sources such as tuning-forks or organ pipes. The frequencies of the two sources should differ by not more than about 10 c.p.s. The intensity of the sound is then heard to pulsate. If the frequency of the beats is too high the ear fails to distinguish the individual pulsations and hears them as a separate note. It is instructive to experiment with two identical tuning-forks whose frequencies can be altered slightly by moving weights up and down the prongs (Fig. 420) or simply by adding varying amounts of wax to the prongs. The difference of the frequencies of the two forks for any given positions of the weights can be determined by timing the beats.

The frequency of an unknown tuning-fork can be found if it is sufficiently near to that of a standard fork to produce beats which can be counted. By doing this the difference of the frequencies is obtained, and in order to decide whether the unknown frequency is equal to that of the standard fork *plus* or *minus* the frequency of the beats, it is necessary to test the effect on the beat frequency of a gradually increasing load of wax on the prongs of the unknown fork. If this fork had the higher frequency before loading, the initial effect of the progressive loading will be to diminish the beat frequency to zero, after which it will increase continuously. If the unloaded fork already had a lower frequency than the standard, the loading will merely cause a continuous increase of beat frequency. Two sources of sound can be tuned to the same frequency very accurately by the elimination of beats between them.

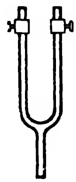


FIG. 420

The production of sound waves of adjustable frequency by a loudspeaker operated by a radio circuit is often achieved by the **heterodyne** or **beat frequency** principle. The arrangement consists of two independent circuits each producing alternating currents whose frequencies are above the limit detectable by the ear. By adjusting one of the frequencies, beats of any desired frequency can be obtained.

2. HUYGENS' CONSTRUCTION. RECTILINEAR PROPAGATION. DIFFRACTION

Wave-Front.—Before proceeding to a further discussion of the behaviour of sound waves it is useful to consider some aspects of wave propagation which have not so far been mentioned.

When a disturbance is initiated by a small source it spreads outwards through the surrounding medium and, after a given time, will have reached a definite distance in any chosen direction. It is quite possible for the medium to have such properties that the speed of the disturbance varies with the direction of propagation. The surface passing through all the simultaneous positions of the disturbance is called a **wave-front.**

More precisely we may say that a wave-front is a continuous surface passing through particles which are in the same phase of their oscillation.

By way of illustration let S in Fig. 421 (i) be a point source of waves in a medium in which the speed of propagation is the same in every direction. The wave-fronts are then concentric spheres with S as centre and the waves are said to be **spherical waves.** In a two-dimensional drawing such as Fig. 421 they are represented by concentric circles. In Fig. 421 (ii) the velocity of the disturbance originating at S varies with direction. It is quite common for this to occur, *e.g.* with light waves in certain crystalline media, and the wave-fronts may then be ellipsoidal. They are represented by ellipses in the drawing.

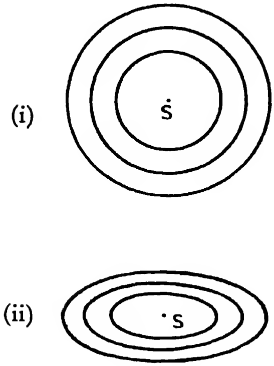


FIG. 421

When a spherical wave has travelled a great distance from its source the wave-fronts have large radii, so that a small portion of any one of them is sensibly plane. We then have **plane waves.**

Huygens' Principle.—In Fig. 422 a particle which is oscillating about O is giving rise to a transverse wave (indicated by the full line) which at the instant under consideration has reached the point B. At a later time the wave has reached B′ and its outline is then shown by the broken line. The additional portion of the wave between A′ and C has been caused by the movement of the particle from A to C. We can similarly regard the disturbance which now exists beyond B, namely the portion of the wave EB′, as being due to the oscillation of the particle whose undisturbed position is B. This particle has moved from B up to D and down again

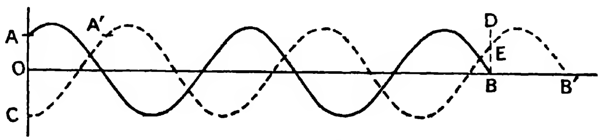


FIG. 422

to E. Thus any chosen particle in a medium through which a wave is travelling may be regarded as an oscillating secondary source contributing to the disturbance which exists beyond the particle.

Of course, if a single particle in an otherwise undisturbed medium is made to perform an oscillation it will send a wave in all directions. The particle at B in Fig. 422, however, does not send a wave backwards similar to the one which is propagated forwards because the particles behind it are not initially undisturbed as those in front of it are.

The idea of wave propagation by secondary sources is embodied in a principle put forward by Huygens in connection with the propagation of light, which we can explain by examples as follows. In Fig. 423 (i) the arc PQ represents the instantaneous position of a spherical wave-front whose source is S. We regard every point on PQ as a secondary source of wavelets and, since the original wave is spherical, the speed of propagation through the medium must be uniform in all directions, so that the wavelets themselves will be spherical. Portions of some of these secondary wave-fronts originating from the marked points on PQ are drawn on the side of PQ farther from S. According to Huygens' principle, the surface P′Q′ which touches the secondary wave-fronts is the new position of the wave-front which was previously at PQ. The subsequent propagation of

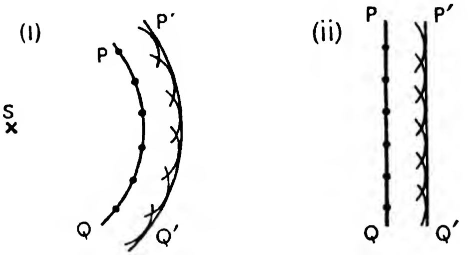


Fig. 423

the wave can be represented by successive repetitions of the same construction. Fig. 423 (ii) shows the propagation of a plane wave in terms of Huygens' construction.

It might be thought at first sight that the use of Huygens' construction is an unnecessary complication in the two examples given because the successive positions of the wave-front can readily be determined without its help. We shall see, however, that it is of great value in discussing reflection, refraction and diffraction of waves.

Fresnel's Zones.—The idea of secondary wavelets implies that every point on a wave-front contributes to the disturbance which eventually arrives at any particular point P towards which the wave-front is moving. In order to discuss the contributions of the various regions of the wave-front and to assess their total effect at P, we make use of the principle of half-period zones which was introduced by Fresnel to account for the behaviour of light waves. We shall give only a brief account of these zones here. A fuller treatment is contained in Chapter LII (Vol. 4) on the diffraction of light.

Suppose that a plane wave of wave-length λ is advancing towards P (Fig. 424). We imagine a plane (indicated by the dotted outline) drawn parallel to the wave-fronts, *i.e.* perpendicular to the direction of propagation OP. We now consider the propagation of secondary wavelets from the plane to P. Since the distance from the plane to P increases

from the point O outwards, the waves arriving simultaneously at P have started from the various points in the plane at different times and are therefore in different phases. For example, the waves which arrive simultaneously at P from points whose distances from P differ by half a wave-length would have started from the plane at times differing by half a period. They would therefore be in opposite phase and would tend to cancel each other at P.

The estimation of the total effect at P can be made by dividing the plane into Fresnel's half-period zones by circles, as shown in Fig. 424, whose radii are such that $(AP)-(OP)=\frac{\lambda}{2}$, $(BP)-(AP)=\frac{\lambda}{2}$, and so on. The zones are the annular regions between the circumferences of the circles, the innermost zone being the complete circle of radius OA. Wavelets from points in any given zone tend to cancel those from corresponding points in the neighbouring zones on either side since their phases differ by π. The cancellation is not complete however, because the effect of the zones diminishes as their radii increase. One way of summing the effects of all the zones is to proceed on the principle that the effect at P of each zone is cancelled by the combination of half the effects of its inner and outer immediate neighbours. The net effect of the whole plane can then be regarded as the sum of half the effect of the innermost zone (*i.e.* the circle of radius OA) and half that of the outermost zone. The effect of the latter is negligible on account of its remoteness from P if the wave-front is very wide, so that we are left with half the effect of the innermost zone.

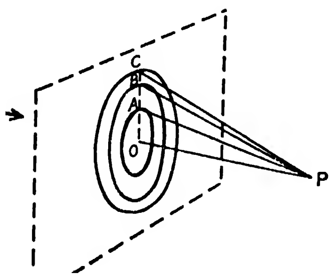


Fig. 424

The radius of the innermost zone, OA, can be shown to be approximately equal to $\sqrt{\lambda(OP)}$. Suppose that OP is 100 cm. Then, in the case of **light,** for which λ is about 6×10^{-5} cm., the radius OA is less than 1 mm. We can therefore regard light as travelling (approximately) by rectilinear propagation, since, although we have supposed that each point on the wave-front sends a disturbance to any given point P ahead of it, yet, as we have seen, destructive interference between the contributions from various points causes a small central portion of the wave-front to be the only effective carrier of the energy.

With **sound** the matter is different. The wave-length (in air) of a note near the middle register of the piano is about 100 cm. For a point at the same distance (100 cm.) ahead of the wave-front as in the light example, the radius of the central zone is therefore of the order of 100 cm. For a case such as this the propagation cannot be regarded as rectilinear.

Shadows. Diffraction.—It must be realized, of course, that if the central part of the wave-front in Fig. 424 is obscured by an obstacle, the remaining zones send waves to P and, by the same argument as is used for the complete unobscured wave-front, the disturbance at P is equal to half the effect of the smallest unobscured zone, *i.e.* the zone which begins at the edge of the obstacle. Thus, for example, if an opaque circular disc covers the circle of radius OB (Fig. 424), the disturbance at P will be due to half the contribution from the ring BC. Evidently, since this statement implies a bending or **diffraction** of the waves round the edges of the obstacle and into the geometrical shadow, it is not in accordance with the principle of rectilinear propagation or with the commonly observed behaviour of light. It obviously corresponds with our experience of sound waves however, because we do not normally observe the complete cutting off of sound by an obstacle. As the size of the obstacle is increased, the intensity of the sound in the central region of the shadow diminishes because the obliquity of the effective zone round the edge of the obstacle increases. Thus, other things being equal, large obstacles produce more complete screening than small obstacles at any particular point. Similarly, for a given obstacle, the screening is more complete and the shadow is sharper when the wave-length is small, *i.e.* when the frequency is high. The zones are then small and a large number of them are obscured by the obstacle. An often quoted example of the effect of wave-length on the sharpness of sound shadows is the case of a moving locomotive which passes behind a building or wall situated between it and the observer. A sharp and perhaps complete cut-off of the high-frequency hiss of the steam is noticed, while the other lower pitched sounds associated with the movement of the engine remain audible.

In the case of light the wave-length is so short that the diffraction of light round the edge of an obstacle and into the geometrical shadow is a very small effect and usually passes unnoticed. The phenomenon exists, however, and is both qualitatively and quantitatively in accordance with Fresnel's theory.

3. THE REFLECTION OF SOUND

Echoes. Architectural Acoustics.—A common and obvious example of the reflection of sound waves is the echo. The reflecting surface is sometimes a cliff, a steep hill, or the side of a large building. Provided that the distances are large enough, there is an appreciable time interval between the arrival of the direct and reflected waves at the ear. When the speed of sound in known, echoes may be used for the determination of the distance from the source and observer to the reflecting surface. This principle has been applied to the determination of the depth of the sea immediately below a ship equipped with the necessary apparatus for recording the time interval between the initiation of a sound in the water

and the reception of the wave reflected from the sea-bed. This is called **echo sounding.**

The reflection of sound waves occurs in rooms and halls. In such cases the times involved are usually much shorter than with open-air echoes, and the only effect of reflection in a room of ordinary size is to increase the loudness with which sounds produced in the room are heard. The multiple reflections from walls, ceiling and floor bring more sound energy to the ear than would be the case in the open air. In a large hall, however, the time interval between the arrival of the direct and reflected waves may be sufficient to interfere with the clear reception of the sound, and thus to detract from the intelligibility of speech and the quality of music. Evidently the sound of a spoken word or syllable may reach the ear by reflection at the same time as one which was uttered later but has travelled by the direct path.

One of the ways of testing the acoustic properties of a hall is to measure its **reverberation time.** In order to do this, a source of sound such as an organ pipe or a loud-speaker is operated in the room. When the source is suddenly stopped, the reflected waves continue to reach the ear. The reverberation time is defined as the time taken for the sound intensity to fall to one-millionth of its value at the instant when the source was cut off. A pistol shot is also often used as the source. It is found that speech may not be fully intelligible if, owing to the strength and number of successive reflections of sound waves between the bounding surfaces of the room, the reverberation time is more than about $1\frac{1}{2}$ sec.

The reverberation time of a given room can be reduced by covering the walls and ceiling with special sound-absorbent materials so that the intensity of the sound is greatly reduced at each reflection. An audience has an absorbent effect, and this accounts for the improvement which is often observed in the acoustic properties of a hall when a full audience is present. It must not be supposed, however, that the ideal concert hall is one in which the reverberation time is very small. If the hall is too absorbent, music performed in it sounds "dead" and unpleasing.

The Mechanism of Reflection.—Suppose that a plane solid surface is placed at right angles to the direction of propagation of a plane sound wave in a medium. The wave causes periodic condensations and rarefactions to occur in the layer of medium situated in front of the plane surface just as it does at all other points in the medium. This layer therefore acts as a source of sound waves which are propagated back into the medium since they cannot be propagated forward. These constitute the reflected wave. The subsequent state of oscillation throughout the medium is due to the superposition of the incident and reflected waves.

If the reflecting surface is completely rigid and incapable of movement, then the particles of the medium which are immediately in contact with it are also unable to move. Therefore the principle of superposition requires that the displacement of these particles due to the reflected wave must at all

times be equal and opposite to the displacement which they would have if they were disturbed by the incident wave alone. In order that this shall be so, the incident and reflected waves must be identical in all respects (except that their directions of propagation are opposite), and there must be a phase difference of half a vibration, or π radians, between them at the reflector. The state of oscillation which exists in the medium as a result of the simultaneous existence of the two waves has already been discussed in Section 3 of Chapter XXXIV (page 563), where it is shown that two identical waves travelling in opposite directions give rise to stationary waves characterized by nodes and antinodes. The condition that there shall be no vibration at the reflecting wall makes it necessary that every point on the reflector shall be a node. The state of affairs is illustrated in Fig. 425, which is similar to Fig. 411 (page 564). On the

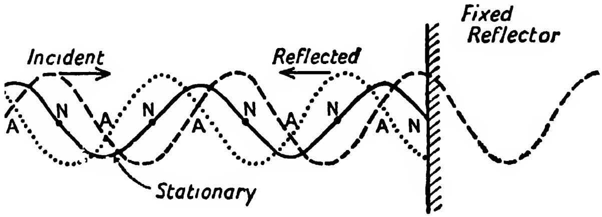


FIG. 425

left of the wall the incident wave is represented by the heavy broken line travelling to the right, while the dotted line indicates the position of the reflected wave at the same instant. Their relative phases are such that the stationary wave (the full line) has a node at the wall. Drawings of the conditions at other instants can be obtained by moving the incident and reflected waves through equal distances to the right and left respectively (see Fig. 411).

It has already been said that the displacement due to the reflected wave at the wall is always equal and opposite to that due to the incident wave, that is to say there is always a phase difference of half a complete vibration, or π radians, between the displacements at the wall due to the two waves. Reflection at a fixed wall is therefore said to involve a change of phase of π. This point is made clear by considering the light broken line (Fig. 425) which represents the continuation of the incident wave on the right-hand side of the wall. If this curve is turned back on itself at the wall we do *not* obtain the reflected wave unless, in addition, the curve is turned upside down. The latter step introduces the phase change of π.

Since the wall is situated at a node, the pressure variation in the medium just in front of it is, unlike the displacement variation, a maximum. As we have just seen, the reflected wave may be regarded as being generated by the pressure variation produced in front of the wall by the incident

wave. As regards pressure, therefore, the two waves are in phase with each other at the fixed wall, or, as it is frequently stated, a condensation is reflected as a condensation and a rarefaction as a rarefaction. Reference to Fig. 425 will show that this is the case. It must be remembered that since we are dealing with longitudinal waves the displacements actually occur to right and left of the mean positions of the particles although they are represented vertically on the graphs. If displacement to the right is represented on the graph by displacement upwards, then the pressure is above normal in regions where the displacement curve slopes downwards towards the right (page 561). Thus at the instant depicted in Fig. 425 the incident wave is causing a condensation in front of the wall, and the reflected wave also slopes downwards towards the right and is therefore also causing a condensation. This similarity of slope exists at all times

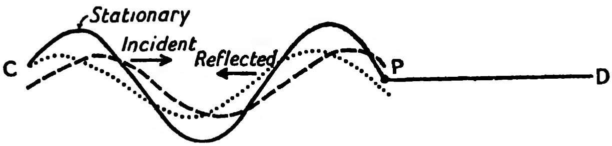


FIG. 426

at the nodes (see Fig. 411, page 564), and at any instant the condensation or rarefaction in front of the reflector is double that due to either of the individual progressive waves. In discussing the phase relation between the incident and reflected waves at the reflector it is obviously necessary to distinguish between pressure and displacement.

A very similar argument to the foregoing applies to the reflection which occurs when a transverse wave travelling on a rope or string reaches a fixed point. Suppose that a wave represented by the broken line in Fig. 426 is travelling to the right on a rope CD which is fixed at P. If P were not fixed, the incident wave would cause this point to vibrate like all the other points between C and D. This means that the portion of the rope immediately to the left of P would be exerting a periodic force on P which, if P were free to move, would cause the wave to be propagated beyond it. When P is fixed, however, this force is exerted on the clamp or other fixing device which, by Newton's third law, exerts on the rope to the left of P an equal and opposite periodic force. This latter force is responsible for the reflected wave. The fixing of P causes the incident wave to be turned back instead of being propagated beyond P. Since the displacement of P is always zero, the reflected wave must be such that the displacement of P due to it alone is always equal and opposite to the displacement which P would have by virtue of the incident wave alone. The two waves are therefore identical; and the system of stationary waves set up on the portion of the rope CP has a node at P. Furthermore, as in the case of

the reflection of a longitudinal wave by a wall, there is, as regards displacement, a phase difference of π between the two waves at the point of reflection.

It should be mentioned that the reflectors we have postulated in the foregoing discussion—the rigid wall and the immovable point on the rope —are *perfect* reflectors. In practice no reflector is perfect. For example a wall, however rigid and massive, will vibrate slightly when sound waves impinge on it, thus allowing some of the incident sound energy to penetrate it—a fact well known to dwellers in flats and small houses. Imperfect or *partial* reflection causes the reflected wave to have a smaller amplitude than the incident wave, with the result that the nodes of the stationary wave are not perfect.

A type of reflection which differs from that due to a fixed reflector occurs when sound waves reach a boundary separating the medium in which they are travelling from one which is less dense. An extreme case of this type of reflection would occur at a surface separating a medium from a vacuum. Suppose that a longitudinal wave is travelling in the medium at right angles to the bounding surface. In a condensation the particles of the medium are moving forward in the direction of propagation and, on account of the enhanced pressure, they are pushing forward the particles in front of them against the resistance due to the presence of particles still further ahead. When a condensation reaches the boundary, the particles on the surface are pushed forward towards the vacuum against no resistance except the cohesion of the medium. The particles on the surface therefore move further in the direction of the wave than they would if they were in the interior. In fact the condensation becomes a rarefaction and travels back as such into the medium. The next incident rarefaction is changed at the boundary into a condensation and so on. Thus a reflected wave is set up, the phase relation between the two waves being the antithesis of that which obtains in the case of reflection from a fixed reflector.

An example of the same mechanical process occurs when the first of a line of coupled railway wagons is pushed towards its neighbour. The buffer springs between the two wagons are compressed, so that the second wagon moves towards the third. The state of "condensation" is thus transmitted along the line until the last wagon is reached. Since this wagon experiences no resistance, the energy which its neighbour has passed on to it causes it to move away until the coupling is stretched. Thus a condition of "rarefaction" has been generated and is then transmitted back along the line of wagons.

Returning to the reflection of sound waves at a boundary between a medium and a vacuum and examining it in the same way as we did the case of reflection at a fixed wall, we can say that whereas the *displacement* was necessarily zero at a fixed reflector the *strain* in the medium is always zero at the boundary in the present case. In order to see the

truth of this, consider an indefinitely thin layer of the medium situated at the boundary. If the next layer of the medium is displaced, say towards the boundary, by a certain amount, then the boundary layer, being free to move and having a very small mass, will immediately move through the same distance so that the relative positions of the layers remain unaltered. There is no strain, therefore, and the displacement curve representing the resultant of the incident and reflected waves must consequently have no *slope* at the boundary. In order that the two slopes shall cancel each other at all times it is necessary, as in other cases of perfect reflection, that the reflected wave shall have the same amplitude as the incident wave. The two together form stationary waves, and their phases are such as to fulfil the required condition at the boundary. Fig. 427 illustrates the state of affairs. The incident wave (represented by the broken line on the left of the boundary) is shown continued into

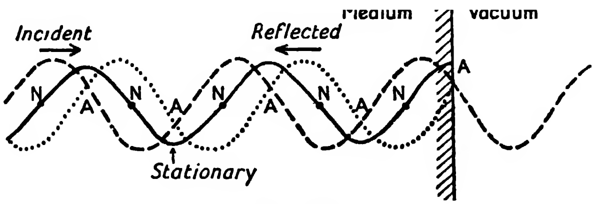


FIG. 427

the vacuum in order to demonstrate that the reflected wave (the dotted line) is obtained by turning the incident wave back on itself *without* inversion. Therefore, unlike the case of reflection at a fixed reflector, there is no change of phase of the displacement curve on reflection. As we have seen, however, the pressure does change phase. The system of stationary waves which is set up is identical with that due to a fixed reflector, except that in the present case the reflector, being a plane of zero strain (but maximum displacement) is situated at an antinode and not at a node. This is the only difference between Figs. 425 and 427.

We shall have to return to the process of reflection in discussing the vibration of gas in pipes. Obviously at the closed or "stopped" end of a pipe the reflection of a sound wave travelling in the pipe is of the former type just discussed—a condensation is reflected as a condensation. At an open end, however, the reflection is of the latter type.

When a sound wave travelling in a solid rod reaches a free end, it is reflected without change of phase and the free end is an antinode.

Sound waves travelling in the atmosphere are partially reflected at a boundary between two regions of differing density. For example, a sheet of hot air rising from a heated horizontal straight tube acts as a sound "mirror." On a larger scale the prolonged sound of thunder is produced by reflections from the surfaces of clouds and other air masses of abnormal density.

Reflection when Incidence is not Normal.—We have so far discussed the effects of reflection when the direction of propagation of the incident wave is normal to the reflecting surface. In such cases the direction of the reflected wave is also normal to the reflector. For plane waves both the incident and the reflected wave fronts are parallel to the reflector. The application of Huygens' principle to this type of reflection is very simple. All points on a given incident wave-front reach the reflector at the same time. Here they give rise to secondary wavelets which are propagated backwards, since the reflecting surface prevents forward propagation. At any given time after the arrival of the incident wave-front the secondary wavelets all have equal radii, so that the reflected wave-front (AB in Fig. 428) is plane and is parallel to the incident wave-front and to the reflector.

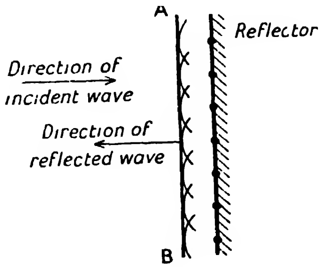


Fig. 428

We shall now briefly mention the application of Huygens' principle to the case of a plane wave incident obliquely on a reflecting surface. A fuller treatment is given in connection with light in Chapter XLI (Vol. 4). Fig. 429 shows a plane wave, represented by the wave-front AB incident obliquely on a reflecting surface RR′. The parallel straight lines CA and DE, which are perpendicular to the wave-front, represent the direction of propagation. They correspond to what are called rays in

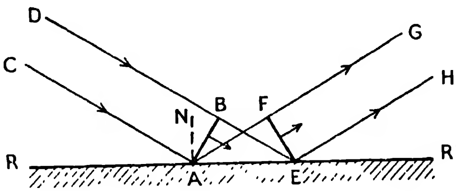


Fig. 429

connection with the propagation of light. It is shown in Chapter XLI that Huygens' principle leads to the experimentally established law of reflection, namely, that the reflected wave-front EF makes the same angle with the reflector RR′ as does the incident wave-front AB. This is usually expressed by stating that corresponding reflected and incident rays (*e.g.* AG and CA) make equal angles with the normal (AN) at the point of incidence.

In Hebb's method of determining the speed of sound in air (page 576) the parabolic reflectors reflect the sound waves according to the above law exactly like the reflection of light by polished metal or glass surfaces of similar shape.

4. THE REFRACTION OF SOUND

It has already been explained that reflected waves are produced at the boundary of two media having different properties. In general (and especially in the case of sound) some of the wave motion incident on a boundary passes into the second medium. Having done so, it continues to travel through the second medium with a speed determined by the properties of that medium.

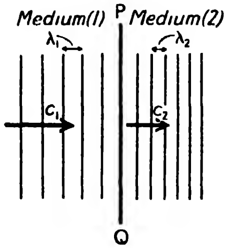


FIG. 430

In Fig. 430, PQ represents the surface of separation between medium (1) and medium (2). A plane wave is travelling in medium (1), and is represented in the diagram by the equally spaced lines lying to the left of PQ and parallel to it. Suppose that the frequency of the waves is n and that their wave-lengths and speeds in the two media are respectively λ_1, c_1 and λ_2, c_2. Let the consecutive incident wave-fronts which are drawn in the figure be separated by distances λ_1. Particles on the boundary PQ will be caused to vibrate by the incident wave. The disturbances due to these vibrations are propagated backwards as a reflected wave and also forwards into medium (2). It is evident that the frequencies of the incident, reflected and transmitted waves are all equal to n. We have therefore

$$n=\frac{c_1}{\lambda_1}=\frac{c_2}{\lambda_2}$$

or

$$\frac{\lambda_1}{\lambda_2}=\frac{c_1}{c_2}$$

Thus the wave-lengths of the waves in medium (1) and medium (2) are proportional to the speeds of propagation in the two media.

Refraction at Oblique Incidence.—When a wave strikes obliquely the boundary between the medium in which it is travelling and a second medium in which the speed of propagation is different, then that portion of the wave motion which passes into the second medium travels in a different direction from that of the incident wave. This change of direction is called **refraction.** Its treatment in terms of Huygens' principle is included in Chapter XLI (Vol. 4) in connection with light and we do little more here than quote the law which governs the phenomenon.

In Fig. 431 the parallel lines CA and DF represent the direction of a plane wave incident on the surface RR′. A wave-front is represented by AB and the two dotted lines are inserted as a reminder that, in general, reflection also takes place. The direction of the refracted wave in medium (2) is indicated by AG and FH, and EF is a refracted wave-front. An application of Huygens' principle shows that if i_1 is the angle

which the wave-front in medium (1) makes with the surface RR′ and i_2 is the corresponding angle in medium (2), then

$$\frac{\sin i_1}{\sin i_2} = \frac{c_1}{c_2}$$

where c_1 and c_2 are respectively the speeds of propagation in the two media. Thus, for a given pair of media the ratio of $\sin i_1$ to $\sin i_2$ is a constant, which is called the **refractive index** of the second medium relative to the first for the particular wave motion concerned. Fig. 431 illustrates a case in which c_2 is less than c_1, and it will be seen that the change of direction takes place in a direction *towards* the normal to the surface at the point of incidence. If c_2 is the greater the refraction takes place away from the normal.

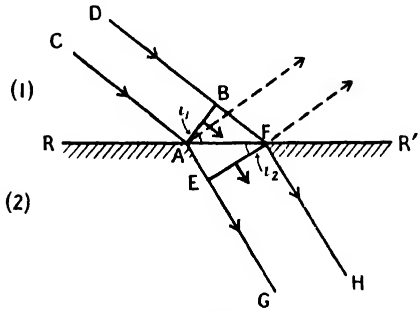


FIG. 431

Refraction by Wind.—It is possible for wind to produce a kind of refraction of sound waves. In Fig. 432 (i) the equally spaced vertical lines represent the wave-fronts of a plane sound wave travelling parallel to the ground in still air. A horizontal air current parallel or anti-parallel to the direction of propagation would have no effect on the direction of

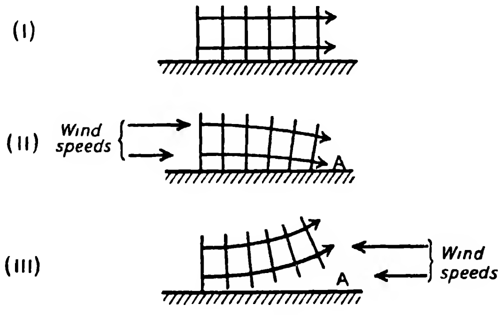


FIG. 432

the waves (although it would affect their speed) unless the wind velocity varies with distance above the surface of the ground. The usual type of wind-velocity distribution is an increase of wind velocity with distance above the ground. The effect of such a velocity gradient when the wind is blowing in the same direction as the waves are travelling is shown in Fig. 432 (ii). The tops of the wave-fronts travel faster than the bottoms, with the result that they lean over forwards as they move. The rays are

therefore curved downwards, with the result that in a region such as A, near the ground, there is a greater concentration of sound energy than there is when the air is still.

In Fig. 432 (iii) the wind is blowing against the direction of propagation and, since the wind speed is greater at greater heights, the tops of the wave-fronts are retarded more than the bottoms. They therefore lean backwards and the rays curve upwards. Audibility is consequently diminished at A. Fig. 432 (iii) explains why a sportsman approaches a bird on the ground "up wind," *i.e.* by walking *against* the wind. A bird at A in Fig. 432 (iii) is less likely to hear the sound of a man's footsteps than it is at A in Fig. 432 (ii).

The drawings in Fig. 432 also serve to illustrate an effect due to a vertical temperature gradient in still air. On a hot sunny day, when the ground is warm, air temperature diminishes with height above the ground, and the speed of sound decreases with height correspondingly (page 573). Therefore a sound wave travelling horizontally will be distorted as in Fig. 432 (iii) and audibility at ground-level will be impaired. If the temperature gradient is inverted, however, as it may well be in the evening when the ground has cooled more than the atmosphere, the conditions shown in Fig. 432 (ii) would obtain, audibility at ground-level being enhanced. This effect is often noticeable on a still evening following a hot day.

EXAMPLES XXXVI

1. Explain the production of echoes, and describe how the phenomenon involved may be studied experimentally in the laboratory.

An observer, standing some distance from a cliff, makes a sharp sound by hitting two stones together. The time interval between the sound and its echo is $3\frac{1}{2}$ sec. After walking 280 yd. nearer to the cliff the observer finds that the time interval is 2 sec. Calculate (*a*) the velocity of sound and (*b*) the observer's original distance from the cliff. (L.I.)

2. How is the audibility of distant sounds affected by temperature and wind?

A sharp sound is produced in front of a flight of steps each 1 ft. wide. Explain the nature of the echo heard by the person making the sound, and find its frequency if the temperature of the air is 20° C. and the velocity of sound in air at 0° C. is 1089 ft. per sec. (L.I.)

3. What are the characteristics of the waves by which sound is propagated in air and upon what factors does their speed depend?

Explain what occurs when (*a*) two identical trains of sound waves moving in opposite directions are superimposed, (*b*) two sound waves of slightly different frequency arrive simultaneously at the ear. (L.Med.)

4. What are "beats"? How are they produced?

Thirty-three tuning-forks arranged in order of increasing pitch are such that any two neighbouring forks give four beats per second and the last fork gives the octave of the first. What are their frequencies? (L.Med.)

5. Give an account of phenomena, arising from the reflection and refraction of sound, which occur outside the laboratory. In the case of refraction consider in particular the effects of wind and temperature. (L.I.)

6. Describe a method for finding the velocity of sound in air. Explain the effect on the audibility of a distant sound of change of air temperature with height. (L.I.)

Chapter XXXVII

FREE AND FORCED VIBRATIONS. RESONANCE

1. FREE VIBRATIONS

Undamped Simple Harmonic Motion.—We have already discussed this type of motion sufficiently to make it unnecessary to do more than to recapitulate. Reference to pages 35–39 (Vol. 1) will remind the reader that S.H.M. may be defined as the motion performed by the foot of a perpendicular dropped on to a diameter from a point which is moving round the circumference of a circle with constant speed. The displacement x of the foot of the perpendicular from the centre of the diameter varies with time t according to the equation

$$x = a \cos \omega t$$

where a is the amplitude of the oscillation and is equal to the radius of the circle, ω is the angular velocity of the radius passing through the point which is moving round the circumference and is equal to the frequency of the oscillation multiplied by 2π. The time t in the above equation is reckoned from an instant at which the oscillating point is situated at the extremity of its swing on the side on which x is reckoned as positive. If t is reckoned from an instant at which the point is passing through its central position, the cosine is replaced by a sine.

Adopting the cosine expression for the displacement, we obtain (pages 36–38) for the velocity at any time t the expression

$$-a\omega \sin \omega t$$

and for the acceleration

$$-a\omega^2 \cos \omega t$$

or

$$-\omega^2 x$$

The negative sign in the expression for the acceleration indicates that the direction of the acceleration is always opposite to that of the displacement, *i.e.* it is always directed towards the centre of the oscillation.

In general, the oscillations of a body are simple harmonic when a displacement of the body from its equilibrium position causes a restoring force which is proportional to the displacement. It is shown in Vol. 1 of this book that S.H.M. is performed by a mass oscillating up and down at the end of a vertical spring, by a body hanging by a wire and performing rotational oscillations about the axis of the wire, and, for small amplitudes, by pendulums oscillating under the influence of gravity. The vibrations of a tuning-fork are also simple harmonic. Many sources of sound,

such as musical instruments, exhibit more complicated displacement-time graphs than the simple sine or cosine curves associated with S.H.M. of a given frequency; but, as has already been mentioned (page 551), all such curves can be represented as the sum of a series of correctly chosen sine and cosine components.

Undamped free S.H.M. cannot exist in practice because frictional forces can never be entirely eliminated. In order that the oscillations of a system such as a pendulum or a tuning-fork shall not die down, it is necessary to supply energy at a sufficient rate to compensate for the work which is done against frictional forces. This is usually done by acting on the system with a suitably timed force once during each vibration.

Damped Simple Harmonic Motion.—In order to work out by means of Newton's laws of motion how a body will move under the simultaneous action of a restoring force proportional to its displacement and a frictional force, it is necessary to make an assumption about the way in which the frictional force varies during the motion. The most reasonable way of doing this is to suppose that the frictional force is proportional to the velocity of the body. We know that this is true in the case of bodies moving through a fluid medium (*e.g.* the air), provided that the motion is not so fast as to cause turbulence (page 238, Vol. 1). In many other cases where the retarding force is due to different factors the correspondence between the observed and calculated motion of the body appears to justify the assumption.

Let the mass of the body be m and let the instantaneous values of its displacement, velocity and acceleration be x, v and f respectively. Then, if the restoring force is qx and the resisting force is rv, q and r being constants, an application of Newton's second law of motion gives

$$mf = -qx - rv \quad . \quad . \quad . \quad . \quad (1)$$

It should be remembered that the quantities x, v and f are all vectors, and each must be considered positive when it is in a chosen direction and negative in the opposite direction. The negative signs on the right-hand side of equation (1) express the fact that the restoring and resisting forces oppose the acceleration f. Dividing both sides of the equation by m gives

$$f = -\frac{qx}{m} - \frac{rv}{m}$$

or

$$f = -\omega^2 x - kv \quad . \quad . \quad . \quad . \quad (2)$$

where $\omega^2 = q/m$ and $k = r/m$.

To use the above equation in order to deduce the way in which x varies with t involves integral calculus which is beyond the scope of this book, and we shall confine ourselves to discussing the types of motion which are predicted by the full mathematical treatment.

For the purpose of this discussion it is useful to consider a particular system, and we shall choose one which can easily be set up and examined experimentally. In Fig. 433 two fairly long glass tubes A and B, of about 1 in. diameter, are clamped in a vertical position, and their lower ends are joined by a length of wide rubber tube C on which is fitted a screw clip D. Coloured water or some other liquid of low viscosity is placed in the apparatus and can be caused to oscillate between A and B, through C, by applying pressure to the rubber tube E by the mouth and then releasing the pressure. When the liquid levels in A and B are not the same, the liquid is acted upon by a restoring force proportional to the difference of level, *i.e.* to the displacement of the liquid. At the same time the motion of the liquid is opposed by viscous forces which are proportional to its velocity, provided that the movement of the liquid is not so fast as to be turbulent. The magnitude of the viscous forces can be increased or decreased by closing or opening the screw clip D.

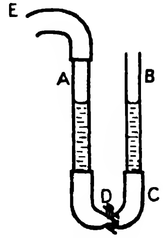


FIG. 433

When the screw clip is open to its full extent and the liquid is released from a displaced position, it oscillates to and fro with a definite time period but with an amplitude which continually decreases, so that the oscillations eventually die away. This type of motion, which is represented by a displacement-time graph such as Fig. 434, is typical of a system in which the resisting force is comparatively small. To be more precise, it applies to all cases governed by equation (2) provided that $k < 2\omega$.

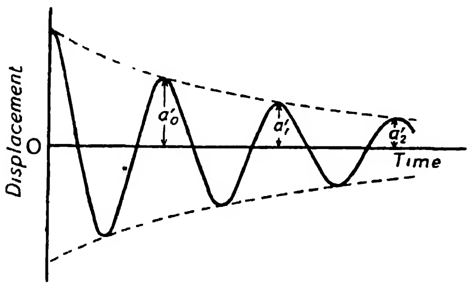


FIG. 434

The variation of the displacement x with time can be represented by an equation of the type

$$x = a' \cos\left\{\left(\sqrt{\omega^2 - \frac{k^2}{4}}\right)t\right\}$$

which shows that the frequency of the oscillation is $\dfrac{\sqrt{\omega^2 - \frac{k^2}{4}}}{2\pi}$ instead of $\dfrac{\omega}{2\pi}$ as it would be if there were no resisting force ($k=0$). The term a' is not constant but represents an amplitude which diminishes with time in an **exponential** manner. This means that during equal intervals of time the *fractional* diminution of a' is the same. Thus, taking successive

complete oscillations as the equal intervals of time, if the maximum displacement is a_0' at a certain instant (Fig. 434) and a_1' is the next value of the maximum displacement on the same side of the zero position (*i.e.* one time period later), a_2' is the displacement after a further complete oscillation and so on, then

$$a_1' = ca_0'$$

$$a_2' = ca_1'$$

$$a_3' = ca_2'$$

where c is a constant fraction equal to the ratio of any maximum displacement to the immediately previous one. Thus

$$c = \frac{a_1'}{a_0'} = \frac{a_2'}{a_1'} = \frac{a_3'}{a_2'}, \text{ etc.}$$

For undamped motion c would be unity, and its value decreases as the damping factor k increases.

If the screw clip is now slightly closed and the liquid in the apparatus in Fig. 433 is set oscillating again, the amplitude will be seen to diminish more rapidly than before; that is to say the value of k has been increased by the increased viscous resistance offered to the flow of the liquid through the constriction in the rubber tube. It should be realized that it is the value of k, the ratio of the resisting force per unit velocity (r) to the mass of the body (m), which determines the rate of decay of the oscillations. Thus free vibrations may be made to persist for a longer time in a system such as a pendulum either by taking steps to reduce r, *e.g.* by making the dimensions of the pendulum bob small in the direction at right angles to its motion through the air (as in clock pendulums), or by increasing the mass of the moving system while keeping r constant or, at any rate, not allowing r to increase by so large a factor as the mass. A frequently quoted example of the principle involved is that of two simple pendulums of the same length, the bob of one consisting of a light hollow table-tennis ball, and that of the other a solid metal ball of the same diameter. The resistance to motion as specified by the factor r is the same for each, because of the equality of the shapes and diameters; but m is very much smaller in the case of the lighter ball, so that k is larger and the oscillations die down more rapidly. The following consideration of elementary mechanics leads us to the same conclusion. If the pendulums are set oscillating with the same initial amplitude, the velocities with which they pass through their lowest positions are initially equal, so that their energies are in the ratio of the masses of their bobs. But the work done against air resistance, *i.e.* the loss of energy during a complete swing of given amplitude, is the same for each pendulum, because the two values of r are equal. Thus the pendulum with the more massive bob, having the greater initial energy, will perform more oscillations than will the other

pendulum. In other words, the rate of decay of amplitude of the lighter pendulum is greater on account of its smaller mass, although it experiences the same air resistance as does the heavier pendulum.

All oscillating systems experience a resisting force of some kind, and their vibrations die out unless they are maintained by the action of an external periodic force. The viscosity of air contributes considerably to this resistance in many cases. A thin quartz rod, fixed at one end and with a small bob attached at the other, will oscillate transversely for a surprisingly long time in an evacuated space, whereas in air at normal pressure its vibrations decay quite quickly. This principle is used to measure low gas pressures.

Non-Oscillatory Motion.—It has already been mentioned that progressive tightening of the screw clip D in the apparatus in Fig. 433 has the effect of making the oscillations die down more and more rapidly. It is now necessary to note, however, that as the rubber-connecting tube becomes more and more constricted, a stage is reached at which the liquid does not oscillate at all no matter how great may be the difference of level from which it is released. The liquid merely subsides towards its equilibrium position at an ever-decreasing speed, and reaches this position with no momentum, with the result that it does not overshoot as it does when the resisting force is smaller. It can be shown that the smallest value of the factor k for which the motion is of this non-oscillatory type is 2ω, and when k has this value the system is said to be **critically damped.** The displacement-time graph is of the character shown by Curve (i) in Fig. 435.

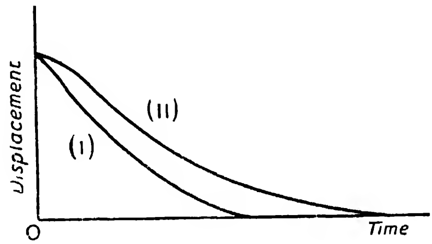


FIG. 435

The motion of the liquid remains non-oscillatory if the connecting tube is constricted to a greater degree than that necessary for critical damping ($k > 2\omega$), but the time taken to reach the equilibrium condition from a given displacement becomes greater (Curve (ii)) and continues to increase indefinitely as the screw clip is tightened. It may be mentioned that the mathematical theory indicates that in all cases an infinite time is necessary for the displacement to fall to zero. In practice, however, the displacement becomes imperceptibly different from zero in a finite time.

FORCED VIBRATIONS. RESONANCE

The Fundamental Equation.—Suppose that a body which is subject to a restoring force proportional to its displacement and to a resisting force proportional to its velocity is, in addition, acted upon by a force

of which the magnitude and sense vary with time according to a sine or cosine law. Such a force would be represented, for example, by the expression $F \sin pt$, where F is a constant (equal to the maximum value of the varying force), and the frequency with which the force varies with time is $p/2\pi$. The motion of the body is then governed by the equation

$$mf = -qx - rv + F \sin pt \quad . \quad . \quad . \quad . \quad (3)$$

or

$$f = -\omega^2 x - kv + \frac{F}{m} \sin pt \quad . \quad . \quad . \quad (4)$$

where the symbols have the same meaning as in equations (1) and (2).

The state of affairs corresponding to this equation of motion may be envisaged by means of the example shown in Fig. 436. A solid block of mass m rests on a horizontal surface and is attached at A to one end of

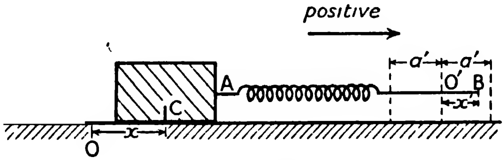


FIG. 436

a horizontal spring, of which the other end (B) is made by some external mechanism to perform S.H.M. horizontally along the axis of the spring with a frequency $p/2\pi$ and an amplitude a'. Suppose that the spring is unstretched, and therefore exerts no force on the body, when the point C on the body is at O and the end of the spring B is at O′. At any instant let the displacement of C from O be x and the displacement of B from O′ be x', each displacement being counted as positive when directed to the right. The force in the positive direction which acts on the body as a result of the displacements of the ends of the spring will be $q(x' - x)$, where q is the force required to extend the spring by unit length. Also if the body is moving to the right with a velocity v at the instant considered, it will experience a frictional force towards the left equal to rv. Therefore, if the acceleration of the body is f in the positive direction, the equation of motion of the body is

$$mf = q(x' - x) - rv$$

Since B is performing S.H.M. of amplitude a' and frequency $p/2\pi$ we can write

$$x' = a' \sin pt$$

and the equation of motion then becomes

$$mf = -qx - rv + qa' \sin pt$$

But qa' is the maximum value of the force exerted on the body as a result of the movement of the end B, because a' is the maximum displacement of B. Therefore qa' is the same as F and the equation of motion is the same as equation (3).

The working out of the relationship between x and t is, as in the case of damped S.H.M., beyond the scope of this book, but we shall quote the important features revealed by the full treatment and by experiment.

It is important to mention that one component of the motion of the body consists of a vibration identical with that which we have already discussed, namely the damped S.H.M. of frequency $\dfrac{\sqrt{\omega^2 - \dfrac{k^2}{4}}}{2\pi}$ which the body would perform in the absence of the applied force $F \sin pt$, *i.e.* if B were fixed at O′. This natural oscillation dies down at the same rate as it would if the applied force were not acting. The other component of the motion is a simple harmonic vibration of frequency equal to that of the applied force ($p/2\pi$) and of constant amplitude. This is called the **forced vibration.** Before the free, or natural, vibration has died out the motion of the body is the resultant of the two vibrations, but after this the forced vibration alone remains.

Damping Neglected.—Before mentioning the effect of damping on the production of forced vibrations we shall examine the hypothetical case in which there is no damping ($k=0$ in equation (4)). This simplification makes a semi-rigorous elementary treatment possible. The equation to be considered is

$$f = -\omega^2 x + \frac{F}{m} \sin pt \quad . \quad . \quad . \quad . \quad (5)$$

and the problem is to find an expression for x, the displacement, in terms of time, which satisfies this equation. We may note that the equation expresses the fact that the actual acceleration f of the body is, at every instant, equal to the sum of the acceleration $-\omega^2 x$ which it possesses by virtue of the restoring force, and the acceleration $\frac{F}{m} \sin pt$ which is due to the applied periodic force. Suppose that, as a result of the action of the two forces upon it, the body performs simple harmonic motion with amplitude a and frequency $p'/2\pi$, that is to say x and t are related by the equation

$$x = a \sin p't$$

Then f, the acceleration of the body at any instant, is equal to $-p'^2 x$, so that

$$f = -p'^2 a \sin p't$$

Equation (5) then becomes

$$-p'^2 a \sin p't = -\omega^2 a \sin p't + \frac{F}{m} \sin pt$$

or

$$(\omega^2 - p'^2) a \sin p't = \frac{F}{m} \sin pt \quad . \quad . \quad . \quad (6)$$

Now if a graph were drawn of $(\omega^2 - p'^2)a \sin pt$ against t and another, on the same axes, of $\left(\frac{F}{m} \sin pt\right)$ against t, each would be a sine wave, and equation (6) is satisfied at all points where the two graphs cross each other. But since the equation must be satisfied for all values of the time t, we reach the conclusion that the two graphs must coincide at *all* points—in fact, that they are one and the same graph. Thus the two frequencies are equal, so that

$$p' = p$$

and the amplitudes are equal, so that

$$(\omega^2 - p'^2) a = \frac{F}{m}$$

or

$$a = \frac{F}{m(\omega^2 - p^2)} \quad \text{since } p' = p$$

The oscillation of the body is therefore represented by the equation

$$x = \frac{F}{m(\omega^2 - p^2)} \sin pt$$

Thus the frequency of the *forced* oscillation is equal to that of the applied force, and its amplitude is equal to

$$\frac{F}{m(\omega^2 - p^2)} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The effect on the amplitude of varying the frequency of the applied force can be discovered from a consideration of the expression (7). If the frequency of the applied force is zero ($p = 0$), the expression for the amplitude becomes $F/m\omega^2$. This case refers, of course, to the steady displacement produced by a constant applied force F. In this case the disturbing force F is opposed by a restoring force amounting to $m\omega^2$ per unit displacement, so that the equilibrium displacement is $F/m\omega^2$. If the frequency of the applied force is now increased from zero, the term $(\omega^2 - p^2)$ decreases, with the result that the amplitude of the forced vibration increases at a growing rate until the expression for the amplitude becomes infinite. This occurs when $p = \omega$, *i.e.* when the frequency of

the applied force is equal to the natural frequency of the system. In practice the inevitable resisting force prevents the amplitude from becoming infinite, but very large oscillations are set up when the damping is small. When the amplitude is a maximum the condition is known as a **resonance.** Everyone is familiar with examples of resonance—for instance the setting up of comparatively large vibrations in a footbridge by suitably timed jumping, the vibration of an article placed on the top of a piano when a certain note is sounded, and the over-emphasis given by an inferior loud-speaker to certain notes which happen to be of the same frequency as its own natural frequencies. We shall discuss other acoustical examples later in this volume.

If the frequency of the applied force is increased beyond that of the natural frequency of the system ($p > \omega$), the quantity $(\omega^2 - p^2)$ begins to increase in numerical magnitude so that the amplitude diminishes continuously until it approaches zero when p becomes infinitely large. We may note also that the expression for the amplitude becomes negative when $p > \omega$. This means that at the instant when $\sin pt$ is equal to unity (so that the applied force has its maximum value (F) in the positive direction of x) the simultaneous maximum value of x is in the negative direction. Thus there is a phase difference of half an oscillation, or π radians, between x and the applied force when $p > \omega$. When $p < \omega$, however, the displacement and the applied force are always in phase.

It is necessary to mention once again that the foregoing discussion ignores the effect of damping. The presence of a resisting force modifies all the above deductions, *including* the condition that the amplitude is a maximum when $p = \omega$. We shall discuss the influence of damping later. Meanwhile it must again be mentioned that in the initial stages of the motion set up by the applied force the natural oscillation of the system of frequency $\omega/2\pi$ is present as well as the forced oscillation. In the complete absence of damping the natural oscillation would never die out, and the resultant motion would always be a combination of the natural and forced vibrations.

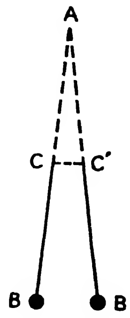


FIG. 437

The characteristics of forced vibrations mentioned above can be illustrated by reference to a simple pendulum. In Fig. 437 let AB and AB′ represent the extreme positions of a pendulum supported at A and oscillating with its own natural frequency. If the string were cut at any point C, the bob and the string CB could be made to oscillate in exactly the same manner as before, *i.e.* between CB and C′B′, by holding the point C and moving it to and fro between C and C′ with the same motion as it performed when the complete pendulum was fixed at A. If this were done, the oscillation of the bob could be described as "forced" because it has a frequency corresponding to a pendulum length AB,

whereas the shortened pendulum, if allowed to swing freely with the point C *fixed*, would have a natural frequency corresponding to the length CB. The state of affairs can be described in terms of equation (5) as follows. In addition to the acceleration ($-\omega^2 x$) which the bob would experience if the shortened pendulum CB oscillated with C fixed, there is a superimposed periodic acceleration $\frac{F}{m}\sin pt$ caused by the motion of C. These two accelerations combine to cause the pendulum of length CB to oscillate in the way it would if it had a length AB and were allowed to swing freely about the point A. Thus when the point of support of a simple pendulum is moved to and fro periodically in the way described, the resulting motion of the pendulum can be discovered graphically as follows. The two extreme positions C and C′ of the point of support are marked, and straight lines AB and AB′ intersecting at A are drawn through C and C′ respectively, each being equal in length to a pendulum which would have the same frequency as the motion imparted to the support. The portions CB and C′B′ are made equal to the actual length of the pendulum. The drawings in Fig. 438 have been constructed in this way, the length of the pendulum and the amplitude of the oscillation of the point of support C being the same in each case. In (i) the frequency of the forced vibration, *i.e.* of the motion of C and of the fictitious pendulum AB, is small compared with the natural frequency of the pendulum CB. In (ii) the forcing frequency is higher than in (i) and the length AB is correspondingly shorter. The amplitude increases as the forcing frequency is raised, and it reaches the maximum possible value when the forcing and natural frequencies are equal (AB = CB) as in case (iii). This is the condition of resonance. In the next case (iv) the frequency with which C is moved to and fro is higher than the natural frequency (AB < CB). The lines CB and C′B′ now cross at A and the motions of C and B are in opposite phase. If the forcing frequency is made higher still, the amplitude of B is further diminished.

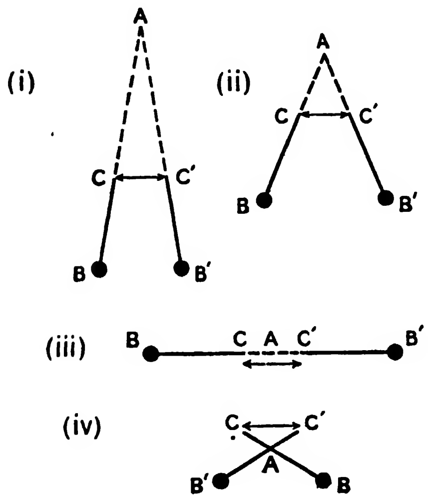


FIG. 438

The two extreme cases are noteworthy. If the forcing frequency is very low indeed, so that AB is very long (Fig. 439 (i)), the amplitude of B is practically equal to that of C. This is understandable, because if

C is moved very slowly, B will follow C without lagging behind and without acquiring a velocity which would cause it to over-swing at the ends of its path. On the other hand, if the rate of vibration of C is so fast that the length AB is practically zero, B will hardly move at all, because no sooner is it pulled in one direction by the string than it is pulled in the opposite direction.

The latter case, corresponding to Fig. 439 (ii), is used in **seismographs** for recording earth tremors. In some forms of these instruments a large mass is suspended in such a way that it forms effectively the bob of a very long pendulum. Its natural frequency is therefore very low, and it fails to oscillate under the action of the comparatively high-frequency oscillations imparted to its supports by the earth movements. A pen which is attached to the large mass traces a line on a moving paper band contained in a mechanism rigidly attached to the earth. Thus the vibrations of the earth are transmitted to the paper but not to the pen, which therefore makes a recording of the tremors.

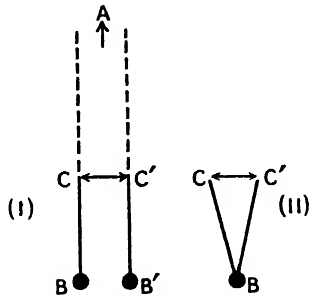


FIG. 439

The Effect of Damping on Forced Vibrations.—When the motion of a body to which a periodic force is applied is subject to a resisting force as well as to a restoring force, the initial natural vibration of the body dies down in the way already mentioned on page 607. After this the forced vibration alone is left. The frequency of this is always the same as that of the applied force, but the way in which the amplitude of the forced vibration depends on the relation between the natural frequency and the frequency of the applied force is modified by the presence of the resisting force. The extent of this modification is illustrated in Fig. 440, in which each of the graphs represents, for a given system (*i.e.* given values of ω and k), the variation of the amplitude of the forced vibration with the value of p expressed in terms of ω, where $p/2\pi$ is the frequency of the applied force and $\omega/2\pi$ is the frequency with which the system would oscillate freely in the absence of both the applied force and the resisting force. The value of k for each curve in Fig. 440 is expressed in terms of ω. The system is critically damped (page 605) when k is equal to 2ω, which means that for this and for greater values of k the system does not initially perform its own natural oscillations. The unmodified forced vibrations are established immediately the external periodic force is applied.

The top curve in Fig. 440 refers to the ideal case when there is no resisting force ($k=0$), which we have already discussed. The peak of the curve, occurring when p is equal to ω, cannot be shown, because the amplitude rises to infinity at this point. It should be noticed that the

amplitude at zero frequency is the same for all curves. . This amplitude is the displacement which would occur if a constant force equal to the maximum value of the periodic force were applied to the system. The resistance has no effect on the displacement in this case since the velocity is zero. At the other extreme, when the applied frequency is very much higher than the natural frequency, the amplitude tends to become zero in every case.

It will be seen from the curves that for a given system (*i.e.* for a given value of ω) one effect of increasing the damping factor k is, as would be

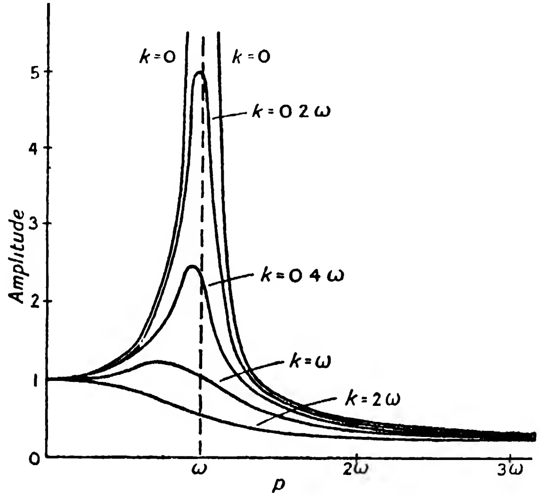


FIG. 440

expected, to lower the maximum amplitude of the forced vibration. The second noticeable effect of increasing the resistance is to flatten the curves, *i.e.* to reduce the height of the graph at the peak more than elsewhere. It can be shown that the peak disappears altogether when k is equal to or greater than $\omega\sqrt{2}$. The resonance is said to be "sharper" when the peak is more pronounced.

A third feature of the curves is the movement of the peak towards the left as the resisting force is increased. It can be shown that for a given value of k the maximum amplitude occurs when

$$p = \quad \omega^2 - \frac{k^2}{2}$$

and this relationship indicates that an increase in k causes a decrease in the value of p for which the amplitude is a maximum.

The phenomenon of the maximum amplitude in Fig. 440 is known

as **amplitude resonance.** It should be clearly remembered that this resonance occurs *neither* at the point where the frequency of the applied force is equal to the natural undamped frequency of the system, *nor* when the applied frequency is equal to the frequency of the damped oscillations. However, when we calculate the velocity v_0 of the body as it passes through its central position, we find that it has its maximum value when p is equal to ω no matter what may be the magnitude of the damping effect. Now, the energy of the body performing the forced vibration is constant with time and is equal to $\frac{1}{2}mv_0^2$, which is its kinetic energy when it is at the centre of its oscillation, since at this place the potential energy is zero. Thus it follows that the *energy* of the forced vibration is at its maximum value when p is equal to ω, no matter what may be the value of k. A series of graphs such as those in Fig. 440, but representing the dependence of energy instead of amplitude upon p, would all have maxima at the same value of p (*viz.* $p=\omega$), the height of the peaks diminishing with increasing damping. The occurrence of this maximum is called **velocity** or **energy resonance.**

It can be shown that for a given value of k, the energy of the forced oscillation when p is equal to, say, $\frac{1}{2}\omega$ is the same as when p is equal to 2ω. Similarly the energies corresponding to values of p of $\frac{1}{3}\omega$ and 3ω are equal to each other and so on. Thus graphs of the energy against p are not symmetrical on either side of the peak, although symmetrical graphs can be constructed as follows. If the frequency of a musical note is twice that of another, the first note is said to be an **octave** above the second, and the second note an octave below the first. For a frequency ratio of 4 : 1 the interval between the notes is two octaves and for 8 : 1 three octaves and so on. Thus if, on the horizontal axis, we mark off a series of equal divisions, each representing one octave, both above and below ω (octaves below being regarded as negative), and then plot the energies corresponding to the values of p represented by these divisions, we find, as already stated, that the energy for the first octave below ω ($p=\frac{1}{2}\omega$) is equal to that for the first octave above ω ($p=2\omega$), and for the second octave below ω the energy is equal to that for the second octave above. Similarly for three octaves ($\frac{1}{8}\omega$ and 8ω) and so on. These curves, which are symmetrical, are shown in Fig. 441.

It will be seen from the figure that, as with amplitude resonance, the curves flatten as the damping factor is increased. This means that the actual energy of the system at resonance is diminished by increased resistance and so also in the the **sharpness** of the resonance, by which is meant the rate of falling off of the energy as we go to left or right of the peak. The effect of resonance is very much less pronounced when the damping is high.

Resonance in mechanical systems such as that envisaged in the above discussion has its counterpart in an electrical circuit carrying an alternating current. The act of "tuning" a radio receiver to the wave-length of a

given transmitter consists in altering the quantity which corresponds to ω. Everyone is familiar with the characteristics of sharp and flat resonance in this connection. When the resonance is flat, the sending station is audible over a considerable range on the wave-length dial and the maximum is not very pronounced. Electrical resistance is responsible for the damping effect in electrical circuits.

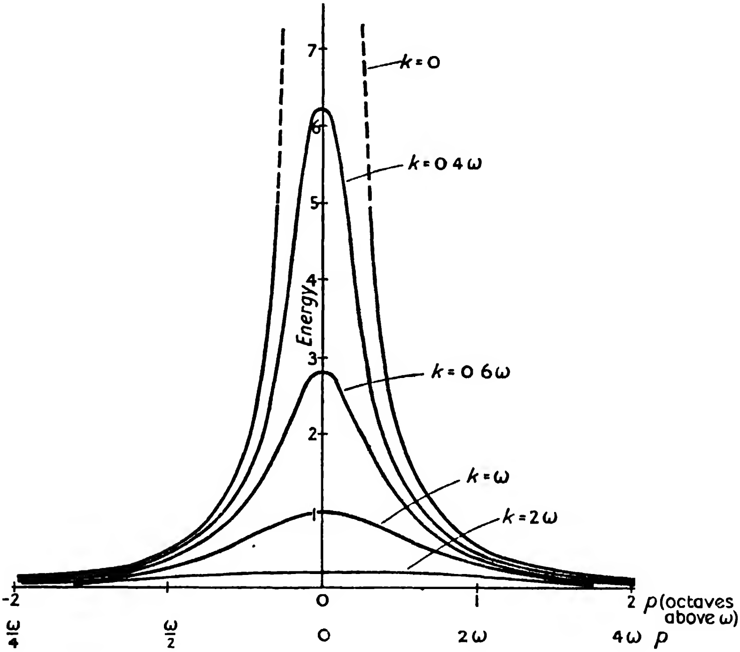


FIG. 441

The fact that the amplitude and energy resonances in a given system do not occur at the same value of the applied frequency may be a little puzzling at first. The explanation lies in the fact that the energy of a body performing S.H.M. is proportional to the square of its amplitude and to the square of its frequency (page 563). Thus, as the frequency of vibration ($p/2\pi$) is increased from the value which gives amplitude resonance, the increase of energy due to rising frequency more than compensates for the decrease of energy due to falling amplitude. Thus the energy continues to rise to a maximum but begins to fall off when the effect of decreasing amplitude becomes greater than that of increasing frequency.

Instructive observations of the properties of forced vibrations can be made by means of a set of pendulums designed by Barton (Fig. 442). A

driving pendulum consisting of a fairly heavy metal weight on a metal rod is attached to a horizontal string tied to two rigid supports. The position of the weight on the rod can be adjusted in order to vary the time period of the master pendulum. The bobs of the pendulums which are to be driven or "forced" consist of inverted paper cones. The threads supporting them have various lengths and are also tied to the horizontal string. The damping of these pendulums is considerable on account of their large effective area and small mass.

To illustrate forced vibrations under conditions of considerable damping, the length of the driving pendulum is adjusted so that its time period is about midway between those of the shortest and longest of the light pendulums. It is then made to swing in a vertical plane at right angles to the plane in which all the pendulums are hanging, thereby causing a to-and-fro motion of the horizontal string and consequently of the points of support of the light pendulums, which immediately begin to show movements. The initial motion of each is the resultant of its own natural vibration, corresponding to its own length, and the forced vibration of the same frequency as the driving pendulum. However, the natural vibration dies down fairly quickly—a separate experiment with the driving pendulum stationary will show the time required for this to occur—and the pendulums are soon performing forced oscillations of the same frequency as the driving pendulum. The amplitudes are seen to vary from small values at each end of the row to a maximum somewhere between. The maximum is not very pronounced on account of the fairly high damping, and its position can be varied by altering the length of the driving pendulum. The oscillation of the pendulums according to the diagrams in Fig. 438 can clearly be seen, each pendulum having the same effective length. The phase of the oscillations varies continuously from one pendulum to the next, the shortest pendulum being most nearly in phase with the driver while the longest pendulum is out of step to the greatest extent. It should be mentioned that the effect of damping is to smooth out the abrupt phase change which occurs when $p=\omega$ in the absence of damping.

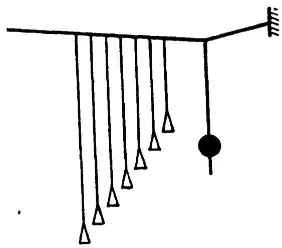


FIG. 442

The effect of damping on the characteristics of forced vibrations and resonance can be studied with Barton's pendulums by placing a wire ring on each of the conical paper bobs. This increases the mass of the bob while keeping the air resistance practically unaltered, so that the factor k in equation (4) (page 606) is decreased. The response of the pendulums is similar to the previous case but the maximum is more pronounced.

Coupled Systems.—Suppose that two pendulums which are identical

in every way are hung from a horizontal string (Fig. 443), and that one of them (A) is made to swing at right angles to the string by drawing it aside and releasing it. In accordance with the foregoing discussion of forced vibrations we should expect the second pendulum (B) to pick up the oscillations because its natural frequency is the same as that of A. This is what actually happens; but when B has acquired a considerable vibration it is just as important to consider the effect of B on A (*i.e.* to regard B as the driver) as it is to think of the matter in the original way.

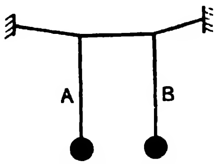


FIG. 443

The process may be discussed in the following terms. The oscillations of B are built up by the action of the periodic force exerted on it by A *via* the horizontal string. According to Newton's third law of motion, the presence of B must necessarily cause A to experience a periodic force which is always equal and opposite to that which B experiences as a result of the motion of A. This reaction on A must be in such a direction as to reduce the amplitude of A, because if it were not, the amplitude, and therefore the energy, of both B and A would be increasing simultaneously, which would be contrary to the principle of the conservation of energy. The amplitude of A, therefore, diminishes while that of B increases, and eventually A is brought to rest. The pendulum B is now the driver and causes A to build up in oscillation, just as its own was generated by A, until B is brought to rest. Thus the process of interchange of energy continues. The two pendulums are described as "coupled," and the rate of exchange of energy, *i.e.* the speed with which each pendulum builds up its oscillations at the expense of the other, depends upon the degree of coupling. This is small when the horizontal supporting string is tight, because under these conditions the motion of the string caused by the oscillation of a pendulum is small.

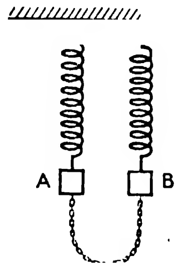


FIG. 444

Another coupled system which provides an illustration of the same principle is shown in Fig. 444. It consists of two similar helical springs supporting masses which are adjusted so that the time period for vertical oscillations is the same in each case. The coupling is provided by the chain attached to each mass. When both the springs are in their positions of rest each supports half the weight of the chain. Suppose that B is initially at rest and that A is caused to oscillate by being pulled down from its position of rest. When this happens the weight of chain which B supports is increased so that B is also stretched, while at the same time the downward force exerted by the chain on A is reduced, which is equivalent to saying that the effective restoring force on A is increased and its downward motion is thereby checked. A similar state of affairs

exists throughout the whole oscillation of A, with the result that its oscillations die down. The weight of the chain governs the degree of coupling.

A certain degree of back reaction on the driver is always present when one system is made to cause oscillations in another, but the periodic stopping and starting of the driving and driven systems which happens when they are identical does not occur if the driver is given a much greater mass than the system which is driven. When this is done, the effects of the reaction on the large energy of the driver can be made imperceptible. This state of affairs prevails in the pendulum experiments of Barton, which have just been described as illustrations of forced vibrations.

The behaviour of a coupled system car be looked upon as a combination of the two possible **modes of vibration** of the system. These are

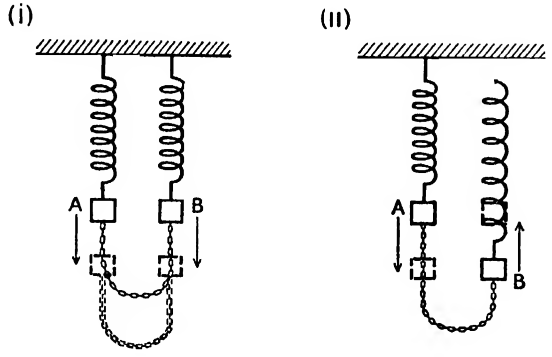


Fig. 445

the free vibrations which the system will perform without showing the gradual transformations which have just been described. To explain this we shall consider the case of the two masses hanging on helical springs and joined by a chain. Suppose that both masses are pulled down to the same extent and simultaneously released. They both oscillate up and down in phase with each other and with equal frequencies without any modification except the inevitable diminution of amplitude due to resistance (Fig. 445 (i)). The frequency will be less than that of either mass and spring vibrating independently because of the additional mass of the chain. Next suppose that one mass is raised and the other is lowered by an equal distance from its position of rest and that they are then simultaneously released. They will then oscillate in opposite phase to each other (*i.e.* with a phase difference of π radians) and no modification of this motion takes place (Fig. 445 (ii)). The lowest point of the chain remains at the same level as the oscillations proceed. Figs. 445 (i) and (ii) therefore represent the two possible modes of free vibration of the system.

In order to obtain the periodic transfer of energy from one spring to the other, they must be started off with a phase difference of $\frac{\pi}{2}$ radians, *i.e.* midway between their two free modes, by holding one mass in its position of rest, raising or lowering the other, and releasing both simultaneously. The subsequent motion, which is described above, is a mixture of the two free modes, and the frequency with which the transfer of motion occurs is found (theoretically and by experiment) to be the difference of the frequencies of the two modes of vibration.

The Helmholtz Resonator.—The name **Helmholtz resonator** is given to a simple acoustical system consisting of a hollow vessel with a small opening which may be simply a hole or a short neck in the wall of the vessel. A commonplace example is a bottle. It is well known that by judiciously blowing across the mouth of a bottle a musical note of definite pitch can be produced. The frequency of the sound is easily shown to depend on the volume of the air in the bottle by introducing varying quantities of water into it. Without going into the mechanism by which the steady jet of air across the mouth of the bottle sets up the vibratory movement responsible for the sound, we can say that the note is produced by the to-and-fro motion of the air in the neck. In order to establish an approximate expression for the natural frequency of vibration of a cavity and neck such as that indicated in Fig. 446, we can suppose that the air in the neck oscillates in and out like a plug or piston.

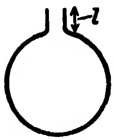


FIG. 446

If it moves inwards it will compress the air in the cavity and will itself experience an outward force opposing its motion, while if it moves outwards the air inside the vessel is rarefied, its pressure falling below that of the external atmosphere, so that the air in the neck is pushed back. Thus, whether the air in the neck moves inwards or outwards, it experiences a restoring force due to the elasticity of the air in the cavity.

Suppose that at a given instant during its oscillation the air in the neck is moved inwards through a small distance x from its position of rest. Regarding the oscillating air as a piston, we can say that this movement entails a decrease of volume of the air in the cavity by an amount Ax, where A is the area of cross-section of the neck. Thus if the cavity has a volume V, the fractional decrease of the volume of the air in it is $\frac{Ax}{V}$. Therefore the movement of the air in the neck has caused the pressure in the vessel to be raised above atmospheric pressure by an amount $\frac{kAx}{V}$, where k is the appropriate bulk modulus of elasticity of the air (page 217, Vol. 1). Consequently, the air in the neck is acted upon by a force $\frac{kA^2x}{V}$ in the outward direction. The mass of this air is $lA\rho$ if the length

of the neck is l and the density of the air is ρ. If the instantaneous acceleration of the air in the neck towards its position of rest is f, we have, by Newton's second law of motion,

$$\frac{kA^2x}{V}=lA\rho f$$

so that

$$\frac{f}{x}=\frac{kA}{Vl\rho}$$

Since the right-hand side of this equation is a constant, the air in the neck performs simple harmonic motion, the frequency (n) being given by

$$\frac{kA}{Vl\rho}=(2\pi n)^2$$

so that

$$n=\frac{1}{2\pi}\sqrt{\frac{kA}{Vl\rho}}$$

$$=\frac{c}{2\pi}\sqrt{\frac{A}{Vl}} \quad . \quad . \quad . \quad . \quad (8)$$

since $\sqrt{\frac{k}{\rho}}$ is equal to c, the speed of sound in air.

Equation (8) is fairly accurate as regards its forecast that n is inversely proportional to $\sqrt{V}$, but the actual value of n given by the equation is found to be in error. The reason for this can be ascribed to the assumption that the length of the oscillating plug of air is equal to that of the neck l. In actual fact some air at each end of the neck shares in the motion, and l in the equation ought really to represent the length of the neck *plus* a correcting term. The necessity for this correction is, of course, more noticeable when the neck is short, and it is especially important when the neck is reduced to a mere hole in the side of the resonator.

The foregoing simple treatment also involves the assumption that, when the air in the neck moves, the consequent change of air pressure in the cavity occurs simultaneously throughout the volume V. This assumption would not be valid if the wave-length of the note emitted were comparable with the linear dimensions of the cavity.

The frequency n given by equation (8) (and corrected where necessary for the effects just mentioned) is the natural frequency of the system calculated by ignoring damping. It is therefore the frequency at which *energy* resonance will occur. That is to say, if tuning-forks of various frequencies are struck and held in turn in front of the aperture of a given Helmholtz resonator, the fork having a frequency equal to n is the one which will cause the loudest response in the resonator, since the loudness of a note depends on the energy of the vibrating source.

A simple laboratory experiment on the properties of a Helmholtz resonator can be performed with a reagent bottle and a series of tuning-forks of known frequency. The volume of water which must be put into the bottle in order that the air in the remaining space may resonate most loudly to each of the forks in turn is determined by running the water in from a burette or graduated cylinder. The volume of the air V which is responsible for the resonance is then calculated for each fork by subtracting the volume of the water actually present from the volume of water necessary to fill the bottle up to the bottom of the neck. The product n^2V will be found to be sensibly constant in accordance with equation (8).

EXAMPLES XXXVII

1. Explain forced vibration and resonance, describing illustrative experiments. (L.I.)

2. A body of mass 500 gm. can move without friction on a horizontal surface. It is attached to one end of a horizontal light helical spring, the other end of which is made to perform S.H.M. along the axis of the spring with an amplitude of 2·0 cm. and a frequency of 15 c.p.s. What is the amplitude of the forced vibration of the body if the spring requires a force of 1 kg. wt. to extend it by 1 cm.?

3. Devise and explain an arrangement other than that in Fig. 436 for the demonstration and investigation of forced vibrations.

4. Explain the importance of damping in a system which is intended to give a faithful record of a varying force. Give examples.

intensity of a note of a given frequency does, however, have a slight effect upon its pitch, but we shall ignore this effect.

Musical Interval.—Two notes which are estimated by ear to have the same pitch are said to be in **unison**—a relationship which can be judged very accurately by a trained musician and to some extent by almost everybody. It is found that, apart from the slight effect which intensity has on pitch, notes which are in unison have the same frequency.

The next simplest pitch relationship is the **octave.** All notes which are represented by the same letter in musical notation are separated from each other by intervals of one or more octaves. Thus, in going up the scale from, say, middle C, the next C above is one octave above middle C. The next C is a further octave higher and is two octaves above middle C.

It is found by experiment that any note which has a pitch an octave above another note has a frequency double that of the lower note. Thus starting with a note of frequency n, the note which is an octave above this has a frequency $2n$, and the next octave above has a frequency $2 \times 2n$, the next octave above this (which is three octaves above the note of frequency n) has a frequency 2^3n, and so on. Thus the frequency ratio for an interval of p octaves is $2^p:1$, and the number of octaves (p) between two notes of frequency n_1 and n_2, the former having the higher frequency, is given by

$$2^p = \frac{n_1}{n_2}$$

or

$$p \log 2 = \log \left(\frac{n_1}{n_2}\right)$$

so that

$$p = \frac{\log n_1 - \log n_2}{\log 2}$$

It is sometimes useful to express a frequency ratio in terms of octaves according to the above formula (*e.g.* page 613). It should be noted that if $n_1 < n_2$ the formula gives a negative value for p. When p is multiplied by 100 the frequency ratio is said to be expressed in centioctaves.

As with the octave, so with all other pitch intervals recognized by musicians, the musical interval between two notes is completely determined by the *ratio* of their frequencies. The *physical* term "interval" is defined as the ratio of frequencies, while the corresponding *musical* interval is denoted by names such as "octave," "major third," "semitone," etc.

If three notes have frequencies respectively of n_1, n_2 and n_3 in ascending order, then the interval between n_1 and n_2 is n_2/n_1 and that between n_2 and n_3 is n_3/n_2. It is obvious therefore that the interval between the first and third (n_3/n_1) is equal to the *product* of the intervals between the

first and second and the second and third. Thus "addition" of intervals is performed by the multiplication of frequency ratios. Conversely the "subtraction" of intervals is carried out by the division of frequency ratios.

Certain combinations of notes when sounded together are pleasing to the ear and are said to be **consonant**, while others are displeasing or **dissonant**. It is often said that two notes are consonant when their frequency ratio can be expressed in terms of fairly low numbers, such as 3 : 2, 4 : 3, etc. However, the absolute pitch of the notes has an effect on their consonance or dissonance. Thus E and C below it (ratio 5/4) when sounded together on the piano produce a much more pleasing sound when they are played in the middle or upper register than they do in the lower register. This and other more detailed observations have led to the belief that the consonance or dissonance between two or more notes is determined by the frequency of the beats (*i.e.* the difference of the individual frequencies) between the notes themselves and between their various overtones. When two notes of equal frequency are sounded together and the frequency of one of them is gradually raised, the beats are first heard as a pulsating of the sound and then as a separate note, the effect of which, combined with the two primary notes, becomes more and more unpleasant until a certain beat frequency is reached. After this, further increase of the beat frequency causes the discord to diminish. The beat frequency for maximum discord varies with the absolute frequencies of the two separate notes.

The Musical Scale.—In western music the range between any note and the octave above it is, in the first place, divided into seven intervals by the insertion of six intermediate notes, the various pitches of which give a number of consonant intervals with each other and with the notes at each extreme of the octave. The following table represents the **natural diatonic scale** in the key of C major, ascending from middle C, which in Physics is usually given the frequency 256 c.p.s., although in music it is 261·2. The notation used is that of Helmholtz, in which $C_1 = 32$, $C = 64$, $c = 128$, $c' = 256$, $c'' = 512$, etc. The tonic sol-fa notation is shown in the second line. The third line of the table gives actual frequencies of the notes, while the fourth line is the series of lowest whole numbers to which the frequencies of the notes of the scale are proportional. The frequency ratios between each note and the initial tonic note are given in the fifth line, while the last line gives the frequency ratios between consecutive notes.

Musical notation	c′	d′	e′	f′	g′	a′	b′	c″
Tonic sol-fa notation	*doh*	*ray*	*me*	*fah*	*soh*	*lah*	*te*	*doh′*
Frequency .	256	288	320	341	384	427	480	512
Numbers proportional to frequency .	24	27	30	32	36	40	45	48
Interval above c′	1	9/8	5/4	4/3	3/2	5/3	15/8	2
Interval between consecutive notes		9/8	10/9	16/15	9/8	10/9	9/8	16/15

It will be noticed that there are three different intervals between consecutive notes of this scale. The interval 9/8 is known musically as a **major tone,** 10/9 is a **minor tone** and 16/15 is a **limma.** The difference between a major and a minor tone is (by division) 81/80. This very small interval is known as a **comma.** A few of the other intervals between various pairs of notes may be mentioned. From *doh* to *soh*, an interval of 3/2, is a **fifth,** and from *doh* to *fah* (4/3) is a **fourth.** Both these intervals are very consonant. From *doh* to *lah* (5/3) is a **major sixth,** *doh* to *me* (5/4) is a **major third,** and *me* to *soh* (6/5) is a **minor third.** It is evident that some of the intervals exist between other notes in the scale than those mentioned above. For example, from *fah* to *doh'* is a fifth, and from *fah* to *lah* is a major third.

The Equally Tempered Scale.—On a keyboard instrument, such as a piano or organ, the notes designated by the letters A to G are sounded by depressing white keys, and additional black keys are situated between each pair of consecutive white keys which differ from each other by a tone, *e.g.* between C and D, D and E, F and G, etc. Each black key sounds a note which is intermediate in pitch between the white notes on either side of it.

Thus there are five black notes between two consecutive C's, and together with the white notes they form twelve pitch intervals in the octave. Readers with an elementary knowledge of music will know that the scale of C major can be played on a piano by starting with any of the keys which sound C and by playing up or down the keyboard on the white keys only. Furthermore, by the appropriate substitution of black keys for white keys *any* note on the piano can be made the starting-point of a major scale which sounds like (*i.e.* contains the same musical intervals as) the scale of C major, although of course it has a different absolute pitch. For example, the scale of G major necessitates the substitution of F♯ (F sharp, the black note immediately above F) for F, while the scale of F involves B♭ (B flat, the black note immediately below B). Other scales involve greater numbers of sharps or flats. The fact that all the scales sound alike on a keyboard instrument means, of course, that a piece of music can be transposed from one key to another with no resulting difference except that of absolute pitch.

The possibility of transposing from one key to another without a perceptible modification of the intervals between corresponding notes in the scale indicates that the various pitches used on the piano are not actually those given above in the scheme of the natural diatonic scale. For example, the interval between *doh* and *ray* in the natural scale of C is 9/8, whereas the corresponding interval in the scales of D and of G are seen from the table to be 10/9. There are many similar examples, although a given interval is not always changed by transposition in the natural scale. Thus some adjustment of the relative pitches of the notes of the scale is necessary if transposition is to be possible without changing the character

of a melody or harmony, and it is easy to see that this can be achieved by making the twelve intervals in the octave of a keyboard instrument all equal to each other. Since the complete octave is an interval of 2, and the addition of intervals is carried out by multiplying together the ratios which represent them, it follows that on a scale of twelve equal intervals to the octave—the "equally tempered" scale—each interval must be equal to the 12th root of 2, *i.e.* $2^{\frac{1}{12}}$. This is the scale to which a piano is tuned, and the following table shows how it compares with the natural scale. The figures represent the interval between each note and C.

	C	D	E	F	G	A	B	C
Natural scale .	1·000	1·125	1·250	1·333	1·500	1·667	1·875	2·000
Equally tempered scale . .	1·000	1·222	1·260	1·325	1·498	1·682	1·888	2·000

It will be seen that simple intervals like 3/2 and 4/3 are not exactly reproduced in the tempered scale, but the difference between the two scales is never very large. The difference between the major and the minor tones disappears in the tempered scale, each becoming 1·122 instead of 1·125 and 1·111 respectively. The limma, which is the interval between E and F and between B and C on the natural scale, becomes $2^{\frac{1}{12}}$ or 1·059 on the tempered scale. This interval is called a **semitone** on the tempered scale, and is the interval between C and C♯, C♯ and D, and in fact between any two consecutive notes of the scale. On the natural scale there is a definite distinction between C♯ and D♭ and between other similar accidentals, but these distinctions do not exist on the tempered scale. The use of such distinctions in music would make the keyboard very complicated, and although theoretically it would be possible to use them on an instrument of continuously variable pitch such as a violin, the difficulty of execution would be much greater than it is with the equally tempered scale.

Combination Tones.—When two notes of frequencies, say n_1 and n_2, are played simultaneously, the sound which is heard does not consist only of the separate fundamentals or "primes" and their overtones. Combination tones are present in the composite sound and they may be quite intense. They consist of the **difference tone,** which is a note of frequency $(n_1 - n_2)$, the **summation tone** of frequency $(n_1 + n_2)$, and possibly other tones like $(2n_1 - n_2)$, etc. There is, of course, the possibility of the formation of other combination tones between the overtones of the primes. It is rather difficult to decide whether the combination tones are subjective or objective. It can be shown mathematically that if, as seems to be the case, the displacement of the mechanism of the ear is not the same for a rarefaction as it is for an equal condensation, then this asymmetry would cause the difference and summation tones to be produced in the ear itself, *i.e.* subjectively. On the other hand, the tones have been detected in the air by observing the response of suitably tuned resonators,

and it is rather doubtful whether the very strong difference tones sometimes heard can be due to what must necessarily be a fairly small degree of asymmetry in the ear. A common example of a strong difference tone is to be found in the type of whistle consisting of two short barrels which give high-pitched notes of slightly different frequency. Each note separately is somewhat thin and not particularly piercing, but when they are sounded together the characteristic loud buzz of the difference tone gives the whistle its peculiar quality. Low-pitched difference tones are often heard when soprano voices are singing with an interval of a third or a fifth between them. When the interval is a fifth ($\frac{3}{2}$), the difference tone is an octave below the lower of the two notes.

Since the harmonics of a note of fundamental frequency n have frequencies n, $2n$, $3n$ . . ., it is evident that the difference tone produced by each pair of consecutive harmonics has the same pitch as the fundamental. Thus the loudness of the fundamental is enhanced during the process of hearing, and its relative intensity seems to be greater than it actually is. This phenomenon is responsible for the fact that there is a comparatively small loss of balance when music is reproduced by a gramophone or radio set which is insensitive to low notes. Even the complete absence of the fundamental of a low note is made up for by the combination tone due to the harmonics.

When a single note of definite frequency and pure sinusoidal wave-form arrives at the ear, the vibrations of the drum are not sinusoidal on account of the asymmetrical restoring force. The sound which actually enters the middle ear is, therefore, equivalent to a series of harmonics of frequencies 1, 2, 3 . . . times the frequency of the original pure note, which, of course, cannot itself be analysed into harmonic components. These **aural harmonics** can actually be heard when a pure note is being received by the ear. Their existence can be demonstrated even more convincingly by varying the frequency of a second pure note while the first one is being heard. Whenever the frequency of the variable note approaches that of one of the aural harmonics, beats are heard. Aural harmonics have no existence outside the ear and are therefore entirely subjective.

3. INTENSITY AND LOUDNESS

The Logarithmic Scale of Intensity.—It has already been stated (page 563) that the intensity of a sound at any place is measured by the rate of flow of sound energy across unit area placed perpendicular to the direction of propagation. Thus intensity is a purely physical quantity and should not be confused with loudness, which is subjective and depends upon estimates made by actual listening. The maximum sound intensity which can be tolerated without actual pain is about 10^{14} times as great as the minimum audible intensity, and it is convenient to deal with this very large range by adopting a logarithmic scale of intensity, which is defined as follows. If two sounds have intensities of I_1 and I_2, the former

being the larger, then the intensity level of the former sound is said to exceed that of the latter by

$$\log_{10} \frac{I_1}{I_2} \text{ bels}$$

Thus one bel corresponds to an intensity ratio of 10 to 1, 2 bels to a ratio of 100 to 1 and so on. There is therefore a range of about 14 bels between minimum audibility and the onset of pain. This is rather a small number to subdivide, so that the **decibel** is usually used. The intensity level of I_1 above I_2 expressed in decibels (db.) is

$$10 \log_{10} \frac{I_1}{I_2} \text{ db.}$$

Evidently 1 db. corresponds to a ratio of I_1 to I_2 given by

$$10 \log_{10} \frac{I_1}{I_2} = 1$$

which means that

$$\frac{I_1}{I_2} = \text{antilog } 0{\cdot}1$$
$$= 1{\cdot}26$$

Thus 1 db. corresponds to a 26 per cent. change of intensity. This change is very roughly the smallest which can be detected by ear without special arrangements, such as alternate listening to the two sounds without a pause between them.

A ratio of intensities of 2 to 1 $\left(\frac{I_1}{I_2} = 2\right)$ is expressed in db. as

$$10 \log_{10} 2 \text{ db.}$$

which is equal to 3·01 db.

It should be realized that it is the intensity level of one sound relative to another which is expressed on the decibel scale. In any particular set of results a certain intensity may be chosen to which to refer all the other intensities. It is evident that this reference level cannot be zero, otherwise all intensity levels relative to it would be infinite.

The decibel scale is also used for the measurement of the sound energy emitted by a source as well as for the intensity of the sound at a given place.

The Scale of Loudness.—As has been mentioned previously, the loudness of a sound is a subjective matter, and it cannot be measured solely by means of physical apparatus—except in certain experiments in which a microphone is made to imitate the response of the ear by suitable adjustment of the electrical circuit to which it is connected. Normally the act of listening enters into the estimation of loudness, and we should expect the loudness of a given sound to be judged somewhat differently by various observers according to the sensitivity of their ears.

The accepted definition and principle of measurement of loudness is as follows. A pure note (*i.e.* one with a sinusoidal wave-form) of adjustable intensity and of frequency 1000 c.p.s. is produced, *e.g.* by means of a valve oscillator. This source is supposed to be situated directly in front of the observer, who listens with both ears and adjusts the intensity of the standard tone until he judges it to be equally as loud as the sound which is to be measured, the two sounds being heard alternately. The intensity of the standard tone is then measured and expressed as, say, n db. above a fixed reference level, which is taken to be that corresponding to an R.M.S. sound pressure of 0·0002 dyne cm.$^{-2}$, this being the minimum audible sound (threshold intensity) at 1000 c.p.s. for the normal ear. The loudness of the unknown sound is then said to be n **phons.**

It should be emphasized that the intensity level of a sound in db. above its own threshold intensity is not necessarily equal to its loudness in phons, unless the sound happens to be a pure note of frequency 1000, in which case the two are equal by definition. In fact, however, loudness is approximately equal to intensity level above threshold within the frequency range 500 to 10,000 c.p.s. Below 500 the loudness is the larger quantity, the difference increasing with decreasing frequency. Again, if the intensity level of a note is increased by a given amount, say by 10 db., the corresponding increase of loudness in phons varies according to the frequency of the note, being, in general, greater for low frequencies. Thus the low notes of a band are enhanced relatively to the high notes as the band approaches the observer, and they appear to die away more rapidly as it recedes.

The fact that, within the range of the commonly occurring frequencies mentioned above, the loudness of a note is not very different from its intensity level above its own threshold, signifies that the loudness (*i.e.* the response of the ear and brain) is a linear function of the logarithm of the actual intensity of the sound. This exponential relationship is an example of the **Weber-Fechner** law which is roughly applicable to all sensations. Its formulation by Fechner followed Weber's observation on muscular response to the effect that when a weight is held in the hand, the minimum perceptible additional weight is proportional to the weight already present. This means that the sensitivity of the body to a stimulus of this kind becomes less as the stimulus increases. This is also true, of course, in connection with hearing, in which, as indicated by the logarithmic law, the response increases by equal amounts for a *multiplication* of the stimulus by equal factors. Every time the intensity of a 1000 c.p.s. note is, say, doubled its loudness increases by a given increment, namely $10 \log_{10} 2$ phons. Thus the increment of intensity necessary to produce a given increment of loudness becomes larger and larger as the intensity is raised.

The very loudest sounds, such as those due to machinery, pneumatic drills, etc., have a loudness of about 100 phons. In an average street the loudness is about 50 phons, while in a very quiet country neighbourhood the loudness falls to about 10 or 20 phons.

EXAMPLES XXXVIII

1. Describe a form of siren suitable for experimenting on the relation between pitch and frequency. State the results which are obtained from such experiments.

If the frequency of a particular note on a piano is 480, what is that of each of the two neighbouring notes? (In the piano the octave is divided into twelve equal semi-tones.) (L.I.)

2. Explain the conditions which determine the *loudness*, *pitch* and *quality* of a musical note.

How may these qualities be varied in the case of a stretched string vibrating transversely? (L.I.)

3. Explain the meaning of the terms *decibel*, *phon*.

A source of sound emits energy equally in all directions at the rate of 0·5 joule per sec. What is the intensity level at a distance of 10 metres reckoned above a datum of 10^{-16} watts per sq. cm.? (L.I.)

4. Give an account of the acoustical principles employed in the construction of the pianoforte. Describe how you would tune a string into unison with a tuning-fork.

If the frequency of the note A is 440 sec.^{-1}, then the frequency of D on the equally tempered scale is 293·665 sec.^{-1}. Find the number of beats per minute between the octave of A and a note whose frequency is three times that of D, when the two are sounded together. (O.H.S.)

5. Explain the terms *phon* and *bel*.

A full orchestra playing fortissimo has a peak power of 70 watts, a violin played softly 4×10^{-6} watts. What is the corresponding change in the intensity level? Give the result to the nearest decibel. (L.I.)

Chapter XXXIX

THE VIBRATIONS OF VARIOUS SYSTEMS

1. THE TRANSVERSE VIBRATION OF A STRETCHED STRING

The Speed of Propagation of a Transverse Disturbance on a Stretched String.—In the elementary theory of the transverse vibration of stretched strings such as those found in the violin, harp, piano, etc., it is assumed that when the string is disturbed transversely, the only restoring force tending to reduce the disturbance and thus to straighten the string is the tension in the string. Thus the effect of the rigidity of the string or wire is supposed to be negligible compared with the effect of the tension, and the theory is therefore applicable only to tightly stretched thin strings or wires. It is also assumed that the disturbance travels along the string without change of shape. An excellent way of appreciating what is meant by the propagation of a disturbance along a stretched string is to tie one end of a long piece of rope to a fixed support and, holding the other end of the rope in the hand, to shake the hand sideways sharply once or twice. The disturbance produced by the movement of the hand can be seen travelling along the rope. It is reflected at the far end and can be seen and felt when it arrives back at the hand. If a length of rubber tubing filled with lead shot is used instead of a rope, the disturbance travels more slowly.

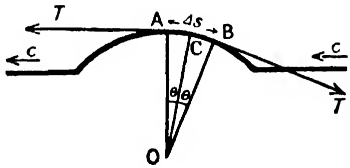


FIG. 448

In order to derive a formula for the speed of propagation (c) of the disturbance, we imagine that the string is made to move contrary to the direction of propagation with a speed c. This would have the effect of making the disturbance stand still with respect to a fixed observer. We then examine the forces acting on a short element of the string AB (Fig. 448) of length Δs. Let the tension in the string be T absolute units of force. Then the element AB is acted upon by two forces T as shown in the figure, each being tangential to the string at its point of application. Let Δs be sufficiently short for it to be regarded as an arc of a circle of centre O. Thus AO and BO are radii of the circle and so is CO, C being the centre of AB. The angles marked θ are obviously equal. The action of the two forces T on the element AB can be obtained

by resolving each force into its components perpendicular and parallel to CO. The two perpendicular components are equal and opposite to each other and so have no effect, while each of the components parallel to CO is equal to $T \sin \theta$ and together they constitute a force of $2T \sin \theta$ in this direction. This force is the centripetal force responsible for the motion of the element AB in the circular path of radius CO. The acceleration of AB in the direction CO is (page 35, Vol. 1) $\frac{c^2}{\mathrm{CO}}$ and the mass of the element is $m\Delta s$, where m is the mass per unit length (or linear density) of the string. Therefore, by Newton's second law of motion,

$$2T \sin \theta = m\Delta s \frac{c^2}{\mathrm{CO}}$$

But $\sin \theta$ is approximately equal to $\frac{1}{2} \cdot \frac{\Delta s}{\mathrm{CO}}$ when θ is small, so that the last equation can be written

$$T = mc^2$$

or

$$c = \sqrt{\frac{T}{m}}$$

It should be remembered that in order that c shall be expressed in cm. sec.$^{-1}$, T must be expressed in dynes and m in gm. cm.$^{-1}$.

The Effect of Reflection at the Fixed Ends.—Suppose that at any instant the string is disturbed in some such way as that indicated in Fig. 449 (i). Even if the disturbance is initially stationary (as is the case with a plucked or struck string) it tends to propagate itself along the string. There is then no more reason why the disturbance should move to the right than to the left, and the result of this symmetry is that a disturbance A_1 (Fig. 449 (ii)) of half the original intensity moves to the left and a similar disturbance B_1 moves to the right. On reaching the end of the string towards which it is travelling each disturbance is reflected in the way described on page 594, the necessary condition being that the reflected and incident disturbances shall cancel each other at the fixed ends P and Q at all times. The type of reflection is, as already explained, characterized by a change of phase of π. The beginnings of the two reflected disturbances A_2 and B_2 are shown in Fig. 449 (iii). The actual disturbance of the string is, of course, the algebraic sum of the incident and reflected disturbances. In Fig. 449 (iv) the reflected disturbances are shown travelling towards each other. In (v) they have crossed over and are approaching the fixed ends of the string, where they are reflected and give rise to A_3 and B_3 (Fig. 449 (vi)). These disturbances have

crossed over in (vii) and have reached the positions of the original disturbances A_1 and B_1 in (ii). Thus a cycle has been completed and would be repeated indefinitely without modification if there were no damping.

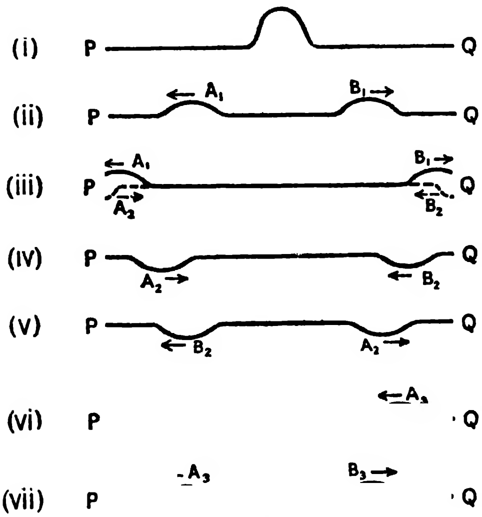


Fig. 449

The cycle is completed in the time necessary for a disturbance to travel a distance equal to twice the length of the string, *i.e.* a time equal to $\frac{c}{2l}$, where l is the length of the string. The frequency with which the string repeats its cycle is therefore $\frac{2l}{c}$.

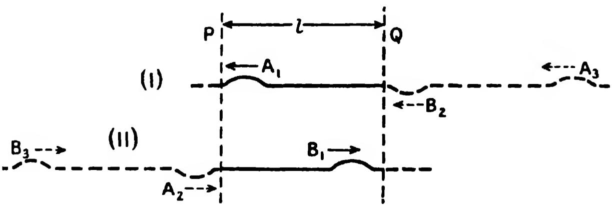


Fig. 450

It will be realized that the change of the state of disturbance of the string with time can be represented by supposing that two equal transverse progressive waves are simultaneously passing along it in opposite directions as indicated in Figs. 450 (i) and (ii), in which each disturbance is labelled to correspond with Fig. 449. The vertical dotted lines indicate the positions of the ends of the actual string, PQ, and it is obvious that the condition

of no displacement at these points is preserved by the simultaneous arrival at each of them of pairs of opposite disturbances like A_1, A_2 and B_1, B_2. It will be noticed also that it is the reflections at the ends of the string which give rise to the equal and opposite progressive waves and not the fact that the original disturbance initially splits into two. One single disturbance travelling along the string will produce the effect of two progressive waves, *e.g.* the A disturbance in Fig. 450.

So far we have said nothing about the shape of the original disturbance on the string, which of course governs the shape, *i.e.* wave-form, of the progressive waves. In accordance with Fourier's theorem, however, each progressive wave can be resolved into a number of sinusoidal waves of appropriate amplitude, and it therefore follows that, no matter what may be the shape of the original disturbance on the string, the resulting vibration of the string can be represented by a series of pairs of progressive sine waves, each pair consisting of two equal waves travelling in opposite directions. The amplitude of each pair of waves will depend upon the character of the initial disturbance. It has been explained previously (page 563) that a pair of identical progressive sine waves gives rise to a stationary wave, and it follows therefore that, in general, a series of stationary waves exists on the string, the shape of the string at any time during its vibration being their resultant. Each stationary wave is subject to the condition that it shall produce no displacement of the fixed ends of the string P and Q. In other words, the only possible stationary waves are those which will fit on to the string with a node at each fixed end. The number of intervening nodes in the case of any particular wave may be anything from zero upwards. This condition is used in the next paragraph to discover the various possible sinusoidal stationary waves which can exist on a string. These are called its **modes of vibration.**

Modes of Vibration of a Stretched String.—The simplest way in which a stretched string can vibrate is called its **fundamental mode of vibration.** In this mode of vibration the nodes at the end of the string are consecutive nodes of the stationary wave, and the state of affairs can be represented by a diagram such as Fig. 451 (i) in which N and N are the two nodes at the fixed ends P and Q, and A is the antinode midway between them. The upper full line, in the shape of a sine curve, represents the position of the string at one extreme of its vibration, and the dotted line of the same shape represents its other extreme position, which it occupies one-half of a complete vibration later. As explained on page 565, the wave-length of a stationary wave is equal to twice the distance between two consecutive nodes, so that the wave-length (λ_0) of the fundamental

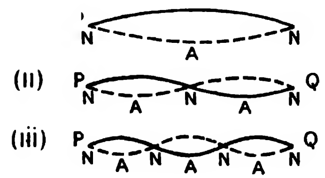


Fig. 451

mode is $2l$. Consequently, the frequency of the fundamental (n_0) is given by

$$n_0 = \frac{c}{\lambda_0}$$

$$= \frac{1}{\lambda_0}\sqrt{\frac{T}{m}}$$

$$= \frac{1}{2l}\sqrt{\frac{T}{m}} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Unless the string is plucked or bowed in an unusual manner, the predominant note emitted is the fundamental whose frequency is given by the above formula. It should be noted that the frequency is inversely proportional to the length and to the square root of the linear density, and is directly proportional to the square root of the tension.

The next mode of vibration which is compatible with the condition that the ends of the string shall be nodes is shown in Fig. 451 (ii). The centre of the string is a node, and the wave-length (λ_1) is l, so that the frequency (n_1) is given by

$$n_1 = \frac{1}{\lambda_1}\sqrt{\frac{T}{m}}$$

$$= \frac{1}{l}\sqrt{\frac{T}{m}}$$

$$= 2n_0$$

The note corresponding to this mode of vibration is the **first overtone,** and is evidently an octave above the fundamental, being twice its frequency.

Next, as in Fig. 451 (iii), we have the second overtone with a wave-length (λ_2) of $\frac{2}{3}l$ and a frequency (n_2) given by

$$n_2 = \frac{1}{\lambda_2}\sqrt{\frac{T}{m}}$$

$$= \frac{1}{\frac{2}{3}l}\sqrt{\frac{T}{m}}$$

$$= 3n_0$$

The next overtone has a frequency of $4n_0$, the next $5n_0$ and so on. Notes with frequencies proportional to the natural numbers 1, 2, 3, 4 . . . are said to constitute a **harmonic series.** The fundamental is the first harmonic, the first overtone is the second harmonic, and so on.

The **quality** or **timbre** of the note from a string (*i.e.* the character which differentiates the sound produced by, for example, bowing from that emitted by the string when it is plucked or when it is struck) is

determined entirely by the number of the possible overtones which are actually present and their relative amplitudes. It requires advanced mathematics to deduce theoretically the relative amplitudes of the overtones when the string is plucked, bowed or struck at a given point, and even then simplifying assumptions have to be made with regard to the initial shape of the string when it is first excited. We can state, however, that overtones which would have a node at the point at which the string is excited are absent from the series. This fact gives the performer on the violin type of instrument control over the quality of the notes emitted, the usual aim being to suppress those overtones which are least consonant with fundamental and the other overtones. When a string is plucked at its midpoint all the even harmonics are absent because they would have nodes at the centre. The lack of these overtones causes the tone to be dull in comparison with that emitted when the string is plucked near one end.

When a string is plucked with a comparatively broad surface such as a finger, more overtones are suppressed than when a sharp plectrum is used, as in playing the mandolin, so that in the latter case the quality is more brilliant, being richer in high-frequency overtones. Mathematical analysis shows that, in general, the amplitude of the overtones produced by plucking diminishes more rapidly with rising frequency than with the other methods of excitation, with the result that plucking produces a more mellow and less brilliant tone than, say, bowing.

Unless it is otherwise stated, the frequency with which a string or wire is said to be vibrating is taken to mean its fundamental frequency.

***Example.**—The tension on a sonometer wire is gradually increased until it starts to beat with a tuning-fork, frequency* 256 *vibrations per sec., until finally there are only* 17 *beats in* 10 *sec. By what percentage of its value must* (i) *the length,* (ii) *the tension, then be altered to bring the wire into unison with the fork?* (L.I.)

Since the tension in the wire was *increased* up to its final value, its frequency was always lower than that of the fork. When the two make 17 beats in 10 sec.—*i.e.* 1·7 beats per sec.—the frequency of the wire is (256 − 1·7) or 254·3 c.p.s.

(i) If the length of the wire is l_1 when it is beating with the fork and l_2 when it is in unison, we have, since frequency is inversely proportional to length,

$$\frac{l_2}{l_1}=\frac{256-1\cdot7}{256}$$

$$\frac{l_1-l_2}{l_1}=\frac{1\cdot7}{256}$$

Therefore the percentage decrease of length which is required, namely $\dfrac{100(l_1-l_2)}{l_1}$, is equal to $\dfrac{170}{256}$, or 0·66 per cent.

(ii) If the tension of the wire is T_1 when its frequency is (256 − 1·7) c.p.s. and T_2 when its frequency is raised to 256, its length being constant, then

$$\sqrt{\frac{T_2}{T_1}}=\frac{256}{256-1\cdot7}$$

Thus
$$\frac{T_2}{T_1}=\frac{256^2}{(256-1\cdot7)^2}$$

$$\frac{T_2-T_1}{T_1}=\frac{256^2-(256-1\cdot7)^2}{(256-1\cdot7)^2}$$

$$=\frac{2\times256\times1\cdot7}{256^2-(2\times256\times1\cdot7)}$$

since we are justified in neglecting $1\cdot7^2$ in comparison with $2\times256\times1\cdot7$. Therefore

$$\frac{T_2-T_1}{T_1}=\frac{2\times1\cdot7}{256-(2\times1\cdot7)}$$

$$=\frac{1\cdot7}{128-1\cdot7}$$

$$=\frac{1\cdot7}{126\cdot3}$$

Therefore the percentage increase in tension, $\frac{100(T_2-T_1)}{T_1}$, is equal to

$$\frac{170}{126\cdot3}$$

$$=1\cdot3 \text{ per cent.}$$

It should be noticed that the above approximate method of calculation saves arithmetical labour.

Experiments with a Sonometer.—A sonometer or **monochord** is a physical, rather than a musical, instrument. A common type is shown in Fig. 452. The "string" is usually a length of steel piano wire fixed to a steel peg R at one end. The wire passes over a freely-running pulley S and the tension is produced by attaching weights to the free end. Usually two fixed bridges (P and U) and one movable bridge (Q) are provided. The length of the vibrating segment PQ can then be adjusted by moving Q. In order to measure the length PQ, it is best to use a metre rule with its edge resting on the tops of the bridges. It is possible for the sonometer to work quite satisfactorily if its base is simply a stout wooden board which acts like the sounding board of a piano and vibrates in sympathy with the string, thereby increasing the intensity of the sound. A louder sound is obtained however if, as in Fig. 452, the bridges are mounted on a hollow wooden box with holes in the sides. In this case the air in the box vibrates as well as the wooden top, and the sound which it produces is emitted into the open air from the holes. The belly of stringed instruments of the violin type acts in this way. The tone of such instruments (*i.e.* the relative intensities of the overtones) depends very much upon the type of wood used and the actual construction of the belly.

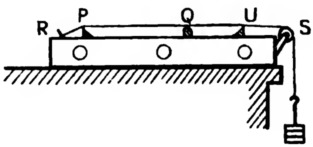


FIG. 452

The fundamental tone of the wire PQ can easily be produced by plucking, bowing or striking with a light hammer. Its frequency is given by equation (1).

It is possible to detect the vibration of a sonometer wire not only by ear but also by hanging on the wire light "riders" such as thin shavings of paper folded into a **V** shape. These are thrown off when the part of the string on which they are hanging vibrates. There are several experiments which illustrate that it is possible for a string to vibrate in a way other than the fundamental mode. For example, the length PQ is made fairly large and the string is struck at its midpoint with a light hammer so that the pitch of the fundamental can be noted. When the note has died away, the midpoint of PQ is held still by resting a pencil lightly on it while one of the halves of PQ is struck at its midpoint. The pencil is then removed and the wire is left vibrating in the mode of the first overtone (Fig. 451 (ii)) with the centre as a node. The pitch of the note emitted is heard to be an octave above the fundamental. Alternatively, the length PQ may be altered until it is in unison with a tuning-fork. Then Q is moved so as to double the length PQ, a paper rider is placed at the centre of PQ and two others are placed midway between the centre and the ends. The fork is then struck smartly and its butt is pressed against the sound box or base of the monochord. The vibrations are transmitted to the wire, and since the frequency of the tuning-fork is equal to that of the first overtone of the wire, the latter resonates and vibrates in this mode. The centre rider, being situated at a node, remains on the wire, while the other two are thrown off because they are at anti-nodes. If the tuning-fork is suddenly stopped by grasping its prongs while it is still vibrating strongly, the wire may be heard to be vibrating with the same frequency. Some sonometers are fitted with two wires, and if this is the case, a length of one of the wires can replace the tuning-fork in the foregoing experiment.

Many experiments with a sonometer involve adjusting the length or tension of the wire until it has the same pitch as a tuning-fork. This can be done in three ways. Firstly, the equality of pitch may be judged by ear, the wire and the fork being sounded simultaneously. This, of course, is easier and more accurate for those who possess a "musical ear," but many non-musical people can learn how to do it satisfactorily by practice. The student of Physics should regard the estimation of equality of pitch as part of his training, and should not abandon the attempt to do it without troubling to practise. Secondly, the principle of resonance may be resorted to as mentioned above. The vibrating fork is pressed on the base or sound box and the natural frequency of the wire is adjusted until paper riders are thrown off. It is necessary to alter the frequency of the wire by quite small steps, *e.g.* by moving the bridge, otherwise the resonance may be missed because it is usually fairly sharp. The resonance of overtones of the wire with the fork must be guarded

against unless specially required. The third method of tuning a wire to a fork is by the elimination of beats. When the fork and the wire are sounding simultaneously and their frequencies are within, say, 10 c.p.s. of each other, beats will be heard having a frequency equal to the difference of the separate frequencies (page 585). When this occurs, the tuning of the wire can be continued in such a way as to reduce the frequency of the beats and eventually to eliminate them altogether. The two notes are now in unison. This method requires a quiet room.

The proportionality relationships embodied in equation (1) may all be verified by means of the sonometer in a variety of ways. The fact that $n \propto \frac{1}{l}$ may be demonstrated by using a convenient fixed tension and tuning the wire to each of a series of tuning-forks of known frequency by altering l. The relationship is then verified by showing that the product nl is a constant, or that the graph of n against $\frac{1}{l}$ is a straight line passing through the origin.

This relationship having been established, we can, if the sonometer is fitted with two wires, keep one of the wires at constant tension and use the reciprocal of its length as a measure of its frequency. Thus to verify that $n \propto \sqrt{T}$ we keep one wire (A) at a fixed length and alter its tension in convenient steps of, say, 0·5 kg. At each step the other wire (B), of fixed tension, is tuned to be in unison with A by altering B's length (l). For each different tension (T) of A its frequency is, by the previously proved relationship, proportional to $\frac{1}{l}$. Therefore, the relation $n \propto \sqrt{T}$ for the wire A is verified by plotting $\frac{1}{l}$ against T and finding a straight line through the origin. Otherwise it may be done by showing that $l\sqrt{T}$ is a constant.

Similarly, to prove that $n \propto \frac{1}{\sqrt{m}}$, we use various wires A of different diameter and material and, keeping their length and tension constant, we tune B to the same note as that emitted by A in each case by altering B's length (l). We know that the frequency of A is then proportional to $\frac{1}{l}$, so that the relationship is proved by showing that $\frac{1}{l} \propto \frac{1}{\sqrt{m}}$ (*i.e.* $l \propto \sqrt{m}$) either graphically, or by showing the constancy of the ratio $\frac{l}{\sqrt{m}}$.

It is evident that a sonometer may be used to determine the frequency of a tuning-fork or other source of sound by tuning the wire to be in unison with the unknown frequency and calculating n from equation (1), T being known from the value of the weights used and m being found by

weighing a known length of wire. If a tuning-fork of known frequency (n) is available, the frequency of another source may be found by altering the length of the wire until it is in unison with the source (l_1) and then with the fork (l_2). The tension and linear density are the same in both cases, so that the unknown frequency is simply equal to $\frac{nl_2}{l_1}$.

Melde's Experiment.—The formation of the nodes and antinodes on a vibrating stretched string can be elegantly demonstrated by an experiment known as Melde's experiment. In this, a stretched thread or fine wire is maintained in oscillation by attaching one end of it to a prong of a tuning-fork. One arrangement which can be used is shown in Fig. 453, in which the fork is held vertically in a heavy stand and the wire is attached to one prong and stretched horizontally over a pulley in a direction at right angles to the plane containing the two prongs (*i.e.* the plane in which the prongs vibrate). If the fork is fairly large (and therefore has a low frequency, say 50 or 100 c.p.s.) its amplitude can be made correspondingly large and the effects are more easily observable. It is also an advantage to maintain the vibration of the fork electrically by an arrangement of the type used in electric bells and buzzers, but it is possible to do the experiment successfully by bowing the fork with a well resined double-bass bow.

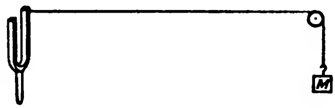


FIG. 453

The experiment consists in attaching a suitable mass M to the far end of the thread or fine wire and moving the pulley along the wire while the fork is vibrating until steady stationary waves are set up. The formation of these waves can be discussed in a similar way to that already used in connection with the vibration of a string which is fixed at both ends. They are due to the combination of incident and reflected waves. In the case of the sonometer wire, waves of any length are possible provided that they fit on to the wire with a node at each fixed end. In Melde's experiment, however, the wave-length is determined by the frequency of the fork and the velocity of the waves $\left(\sqrt{\frac{Mg}{m}}\right)$. Thus when these quantities are fixed, it is necessary to adjust the length of the wire so that the waves will fit on to it, otherwise the wire is not divided sharply into loops. The pulley may be placed at any of the nodes and the standing pattern will always be the same, except, of course, as regards the number of loops between the fork and the pulley. The experiment may also be discussed in terms of resonance. The condition for the formation of a steady stationary wave is that the frequency of the fork shall be equal to that of one of the modes of vibration of the wire so that resonance occurs. By starting with the pulley near the fork and moving it outwards we can, at

the fixed frequency of the fork, cause the wire between the fork and pulley to vibrate in the mode of the fundamental (one loop), the first overtone (two loops) and so on. The distances between successive positions of the pulley will be equal to the length of a loop, which is half a wave-length and is therefore equal to

$$\frac{c}{2n}$$

or

$$\frac{1}{2n}\sqrt{\frac{Mg}{m}}$$

It is possible, of course, to "tune" the wire to the fork by keeping the pulley fixed and altering the weight M.

It might at first be supposed that the end of the wire attached to the fork would be an antinode, since the vibration is, in the first place, generated here. In fact, however, the amplitude of the wire at its antinodes is usually considerably greater than that of the fork, so that the end in question, while not actually being a node, is much nearer to a node than to an antinode.

When the wire has been correctly tuned to the fork, the average distance between consecutive nodes can be measured, and the frequency of the fork can be calculated from the above formula if Mg and m are known. If M is increased by a factor of 4 while the length is kept constant, the wave-length is doubled so that the number of loops on a given length of wire is halved.

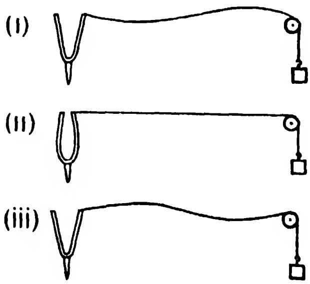


FIG. 454

Another way of performing the experiment is shown in Fig. 454, in which the wire now lies in the plane of the fork instead of at right angles to it. The vibrations are now communicated to the wire in the way shown in the drawings. In (i) and (iii) the string is loose and is able to take up its most displaced shape, but as two such positions of the wire are required for each vibration and the condition occurs only once during each vibration of the fork, the frequency of the wire is half that of the fork. Thus the wave-length is double what it is in the other method of setting up the experiment.

The Frequency of A.C. Supply by Melde's Experiment.—An interesting modification of Melde's experiment can be set up as follows. A fine copper wire is fitted on a sonometer and held in tension by known weights (50 or 100 gm. are convenient weights to begin with). Connections are made as shown in Fig. 455 so as to pass a small alternating current from the mains through the wire. The series resistance R can conveniently be a lamp of suitable power. A strong permanent horseshoe

magnet is placed as shown in the figure, so that the wire passes between its poles N and S. A wire carrying an electric current in a magnetic field of force experiences a force in a direction perpendicular to both the current and to the lines of force, so that in this case the force on the wire is vertical. Its magnitude is proportional to the strength of the current, and its direction is reversed when the current is reversed. The current from the a.c. mains varies in a simple harmonic (sinusoidal) manner with time, and so the wire between the magnetic poles is made to perform S.H.M. in a vertical plane. The frequency of the disturbance on the wire is equal to that of the a.c. mains, and its speed of propagation along the wire is equal to $\sqrt{\frac{T}{m}}$, where T is the weight attached to the wire (in dynes) and m is its mass per unit length. The wave-length is therefore fixed, and stationary waves will be set up provided the length of the wire between the two bridges is an integral number of half wave-lengths, the bridges being nodes. Thus, if T is kept constant the stationary waves can be obtained by adjusting one of the bridges. The wire will vibrate in one or more loops, according to the relative magnitudes of the quantities involved.

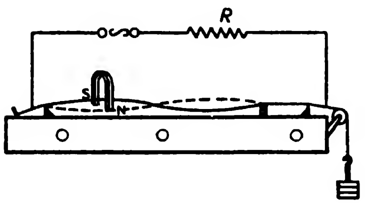


FIG. 455

The mean distance between consecutive nodes is equal to half the wave-length in any particular case. This being known, and m having been determined by weighing a known length of the wire, the frequency n of the a.c. supply can be calculated from the equation

$$n = \frac{1}{\lambda}\sqrt{\frac{T}{m}}$$

From this equation it follows that if various values of T are used and λ is measured for each (after altering the length of the wire to obtain the best possible loops), then a graph of λ against $\sqrt{T}$ will be a straight line whose slope is $\frac{1}{n\sqrt{m}}$.

2. THE VIBRATION OF RODS

Transverse Vibration of Rods.—Fig. 456 (i) shows a flexible rod of, say, steel clamped at its lower end and made to vibrate transversely by pulling its free end to one side and then releasing it. The restoring force causing the vibration is due to the tendency of the rod to straighten itself when bent, which means that the elastic modulus chiefly involved is Young's modulus (page 224, Vol. 1). The clamped end of the rod must

always be a node and the free end an antinode. The vibration shown in Fig. 456 (i) is the fundamental node. The first overtone is shown in (ii) and the second overtone in (iii). The former can be produced fairly easily by holding the rod where the upper node is to appear and striking it sharply about midway between this point and the fixed end. The relation between the various modes is not nearly so simple as in the

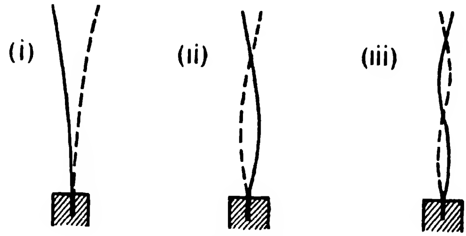


FIG. 456

stretched string. For example, the distance from the free end to the upper node in the first overtone is about 0·23 of the total length of the rod and its frequency is about 6·3 times that of the fundamental, while that of the second overtone is 17·6 times. The theory of such vibrations is much too advanced for this book.

When a rod or bar is free at both ends its simplest mode of transverse vibration involves two nodes as shown in Fig. 457 (i). This makes it possible for the vibrations to be produced experimentally, because the bar can be made to rest on knife-edges at the places where the nodes will appear. The bar can then be made to vibrate by bowing it, and if it is of rectangular cross-section the positions of the nodes can be demonstrated by scattering a quantity of light dust or sand on the top surface of the bar. The dust is driven from the vibrating parts of the bar and collects in sharp transverse lines at the nodes. Theory shows that when the bar is vibrating in its fundamental mode, the distance between each node and the free end to which it is nearest is 0·224 of the total length of the rod. In the first overtone (Fig. 457 (ii)) the nodes are nearer to the free ends and a third node is situated at the centre. The frequency of the first overtone is theoretically 2·76 times that of the fundamental.

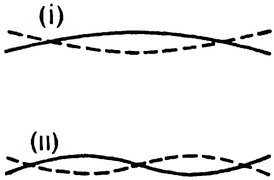


FIG. 457

A similar method of exhibiting the modes of vibration can be used in the case of the transverse vibration of a metal plate. The plate is fixed horizontally by a screw passing through its centre, and can be set into vibration by bowing its edge in a vertical direction. Sand scattered on the surface of the plate collects in lines at the nodes. Many different

patterns, both simple and complicated, can be obtained by bowing at different places while other points on the edge of the plate are held stationary with the fingers. The patterns are known as **Chladni's figures,** and a few examples are shown in Fig. 458.

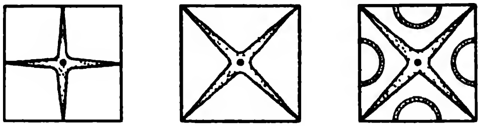


FIG. 458

Vibration of a Tuning-Fork.—It was shown by Chladni that if a free rod is bent at its centre, the two nodes of the fundamental mode approach each other. This is illustrated in Fig. 459. The extreme case where the two halves of the bar are parallel to each other is similar to the case of the tuning-fork. It is evident that when the stem of the tuning-fork is added at the bend it will vibrate up and down in the direction of its axis. This is why a board can be made to vibrate by resting the bottom of the stem of a vibrating tuning-fork on it. When a tuning-fork is struck in the normal way it vibrates in its fundamental mode with a complete absence of overtones, although these can be elicited if the prongs are held at the appropriate places while they are struck with a light hammer.

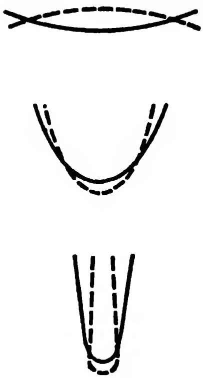


FIG. 459

The Longitudinal Vibration of Rods.—The theory of this type of vibration comes well within our scope. The formula for the speed with which a longitudinal disturbance travels along a uniform rod has already been referred to on page 572. The expression is

$$c = \sqrt{\frac{q}{\rho}}$$

where q is Young's modulus for the material and ρ is its density.

The discussion of the modes of longitudinal vibration of a rod can be presented in a similar manner to that which applies to the transverse vibration of strings, and it is therefore unnecessary to go through it in detail. A small local disturbance such as a longitudinal compression or extension will give rise to two disturbances of half the intensity which travel along the rod in opposite directions and are reflected at its ends. As we have seen in the case of the string, the effect of the successive reflections of a given disturbance first at one end and then at the other is to produce the same state of affairs as would occur if two equal progressive

waves were passing along the rod in opposite directions, each wave consisting of a train of the disturbances separated from each other by a distance equal to twice the length of the rod. The type of reflection which occurs at a free end of a solid rod has already been discussed on page 596, where it is explained that there can be no strain in the solid at the end. This condition causes the reflection to occur without change of phase, and the displacement is always greater at the end than elsewhere. Remembering that the progressive waves can, according to their shape, be regarded as a number of separate sine components, we deduce that the longitudinal vibrations of a bar which is not constrained anywhere can be represented by one or more stationary sinusoidal waves, each of which is subject to the condition that there must be antinodes at each end of the bar. The fundamental mode of vibration is therefore as shown in Fig. 460 (i). The rod extends and contracts periodically while its centre remains stationary. The maximum displacement occurs at the ends. Both ends move towards the centre at the same time, and then both move outwards simultaneously and so on. This is indicated by the arrows in the figure. The wave-length (λ_0) of the stationary wave is $2l$, where l is the length of the rod, so that the frequency of the vibration (n_0) is given by

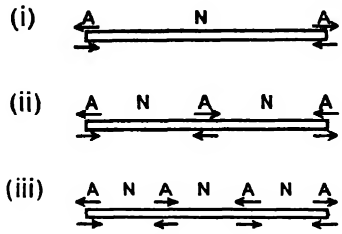


FIG. 460

$$n_0 = \frac{c}{\lambda_0}$$

$$= \frac{1}{2l}\sqrt{\frac{q}{\rho}}$$

As an example, we shall calculate the fundamental frequency of longitudinal vibration of a uniform rod of steel 2 metres long. Young's modulus for steel may be taken as $2{\cdot}0 \times 10^{12}$ dynes cm.$^{-2}$ and its density as approximately 8 gm. cm.$^{-3}$. We then have

$$n_0 = \frac{1}{2 \times 200}\sqrt{\frac{2{\cdot}0 \times 10^{12}}{8}}$$

$$= 1200 \quad \text{approximately}$$

It will be noticed that the frequency is quite high, even for a rod 2 metres long.

The next mode of longitudinal vibration (the first overtone) of a uniform rod which is not constrained anywhere is shown in Fig. 460 (ii). As indicated by the arrows, the displacement at any antinode at any instant is opposite to that at the next antinode. Thus, in contrast with the fundamental mode, the ends vibrate in phase with each other and the centre

vibrates in the opposite phase. The state of vibration is the same as if the two halves of the rod were vibrating in their fundamental modes in opposite phase to each other. The wave-length (λ_1) of the stationary wave is equal to l, the length of the rod, so that the frequency n_1 is given by

$$n_1 = \frac{1}{\lambda_1}\sqrt{\frac{q}{\rho}}$$

$$= \frac{1}{l}\sqrt{\frac{q}{\rho}}$$

$$= 2n_0$$

Thus the pitch of the first overtone is, as in the case of a stretched string, an octave above the fundamental. The next mode of vibration is shown in Fig. 460 (iii). The wave-length is equal to $\frac{2}{3}l$ and the frequency is three times that of the fundamental.

The commonest case of the longitudinal vibration of a rod met with in an elementary laboratory is in connection with the Kundt's tube experiment, which is described later (page 657). In this the rod is clamped at its centre and, with a little practice, it can be set into longitudinal vibration in the fundamental mode by grasping it with a resined cloth or wash-leather and moving the hand slowly from the centre outwards. The rod may be of wood, glass, or metal such as brass or steel. The fact that the rod is clamped at its centre does not affect the fundamental mode because there is a central node in this case. It does, however, eliminate the possibility of the overtones which give an antinode at the centre, *i.e.* the first, third, fifth, etc. As we have seen, plucking a stretched string at its centre suppresses the same overtones because it prevents the formation of the central node which is essential for their existence. The two cases are similar, nodes and antinodes being interchanged.

It is possible, of course, for a rod to be made to vibrate longitudinally with both ends clamped, in which case the ends must be nodes. In the fundamental mode of this system there will be an antinode at the centre, the wave-length will be equal to $2l$, and the frequency the same as when the ends are free and the rod is clamped at the centre. A stretched string or wire will vibrate longitudinally in this way if a resined bow or finger is drawn along it.

Suppose that a wire of radius a cm. is stretched with a tension T dynes between two supports a distance l apart, and let the density and Young's modulus of the material be ρ gm. cm.$^{-3}$ and q dynes cm.$^{-2}$ respectively. The mass per unit length of the wire is equal to $\pi a^2\rho$, so that the fundamental frequency (n_t) for transverse vibration is given by

$$n_t = \frac{1}{2l}\sqrt{\frac{\quad}{\pi a^2\rho}}$$

and the fundamental longitudinal vibration (n_l) is given by

$$n_l = \frac{1}{2l}\sqrt{\frac{q}{\rho}}$$

Therefore

$$\frac{n_l}{n_t} = a\sqrt{\frac{\pi q}{T}}$$

Suppose that the wire is steel ($q = 2{\cdot}0 \times 10^{12}$ dynes cm.$^{-2}$), that its diameter is 0·5 mm. ($a = 0{\cdot}025$ cm.), and that it is stretched by a load of 5 kg. wt. ($T = 5000 \times 980$ dynes). Then

$$\frac{n_l}{n_t} = 0{\cdot}025\sqrt{\frac{\pi \times 2{\cdot}0 \times 10^{12}}{5000 \times 980}}$$

$$= 28{\cdot}3 \quad \text{approximately}$$

This interval is between four and five octaves. Thus if the length of the wire were such as to give a transverse frequency of 256 (middle C), the longitudinal frequency would be about 7250. The unpleasant high notes sometimes unintentionally produced by a beginner on the violin are often due to these longitudinal vibrations.

3. THE VIBRATION OF AIR COLUMNS

Tube Open at Both Ends.—The vibration of air columns can be discussed in a very similar way to that used for strings. In the first place, we can quote the formula $\sqrt{\frac{\gamma p}{\rho}}$ for the speed of sound in air since this was derived on page 569. Next it is necessary to say something about the type of reflection which occurs at the end of a tube containing air. It is obvious that a closed end will cause the same sort of reflection as the fixed end of a string, because the air in contact with the end is incapable of movement. This is described on page 593, where it is shown that as regards displacement there is a change of phase of π, and that a condensation is reflected as a condensation and a rarefaction as a rarefaction. The exactly opposite type of reflection in which there is no change of phase and a condensation is reflected as a rarefaction and *vice versa* is described on page 595, and is shown to be the type which occurs when a wave reaches the boundary between the medium in which it is travelling and (in the extreme case) a vacuum. A state of strain cannot exist at such a boundary because the boundary layer of the medium is subject to no external restraint. These conditions are approximately realized at the open end of the tube, because the air at the end of the tube, being in

contact with the open air, is more free to move than the air within the tube. As explained on page 596, reflection at a boundary which is perfectly free to move gives rise to a stationary wave with an antinode at the boundary. The fact that there is not perfect freedom of movement in the case we are now considering causes the antinode to be situated effectively a short distance outside the pipe. The distance is usually taken to be about $0{\cdot}6r$ in the case of a uniform pipe of circular section of radius r. However, in the discussion which follows we shall not make use of this expression, but shall simply denote the distance by ϵ.

We return now to the discussion of the vibration of the air in the tube. In the case of a stretched string, we began by supposing that there existed on it initially a stationary disturbance, such as that produced by plucking,

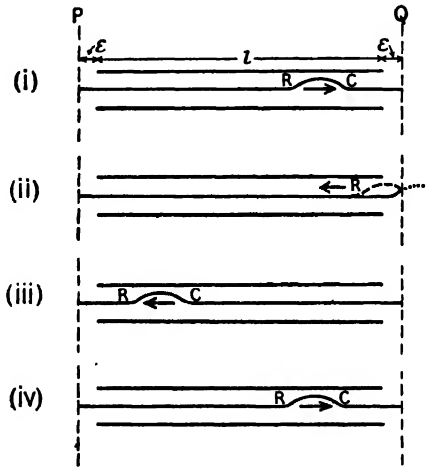


Fig. 461

and we described how, when the string was released, two equal disturbances travelled in opposite directions. This supposition as to the initial conditions which cause the vibrations may be adopted equally well in the case of the tube, but the more usual methods of setting the air into vibration, *e.g.* blowing across one end of the tube or into an attached mouthpiece as in instruments like the whistle or clarinet, provide a source of disturbances at one end. We can therefore start with the assumption that a disturbance of the particles in the longitudinal direction is actually travelling along the tube from this end. It has just been explained that the reflections at the open ends can be regarded as occurring at a short distance ϵ beyond the end of the tube, and that the reflection is of the type which involves no change of phase as regards the displacement of the particles. Fig. 461 (i) shows a disturbance approaching an open end. Although displacements are actually longitudinal they are drawn as

transverse for convenience. Longitudinal displacements to the right are indicated by displacements vertically upwards. On this convention (as explained on page 561) the regions of condensation and rarefaction are situated as shown by the letters C and R in the figure. The dotted lines P and Q at a distance ϵ beyond the ends of the tube, where the antinodes are formed, represent the planes where the reflections can be regarded as taking place. In Fig. 461 (ii) the reflection is occurring according to the conditions already explained. The dotted line on the right of Q shows where the disturbance would have been had it not been reflected, and the broken line on the left of Q is the disturbance which has been reflected without change of phase. The actual displacement of particles on the left of Q is the resultant of the incident and reflected disturbances. Fig. 461 (iii) shows the disturbance travelling back to P after reflection at Q. At P a similar reflection takes place, and in (iv) the same condition as in (i) has been reached. Evidently, therefore, the state of motion of the air in the tube can be represented by two progressive waves travelling in opposite directions as in Fig. 462. The effective length of the tube (PQ) is shown as $(l+2\epsilon)$, the distance between the two vertical dotted lines.

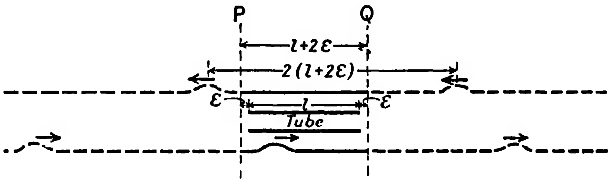


FIG. 462

In Fig. 462 the distance travelled by the disturbance between two identical stages such as (i) and (iv) in Fig. 461 is twice the distance between the planes where reflection occurs, *i.e.* $2(l+2\epsilon)$. This is, therefore, the wave-length of each of the progressive waves, and the frequency with which the air column performs its cycle of vibration is $\dfrac{c}{2(l+2\epsilon)}$.

As in the case of the other vibrating systems we have discussed, we now remind ourselves that whatever may be the shape of the original disturbance the progressive waves can, by Fourier's theorem, be represented by a series of sine waves of appropriate amplitude and frequency. Each of these Fourier components pairs up with the equal sine wave travelling in the opposite direction to form a stationary wave, so that a series of such stationary waves is formed in the air column. The resultant of these represents the vibration of the air. The possible wave-lengths of the Fourier components are determined by the condition that they must produce antinodes at the distance ϵ outside the open ends of the tube. The various possible modes of vibration of the air column are shown in

Fig. 463. The fundamental mode is shown in (i); the upper arrows indicate the direction of motion of the air at the antinodes at a given

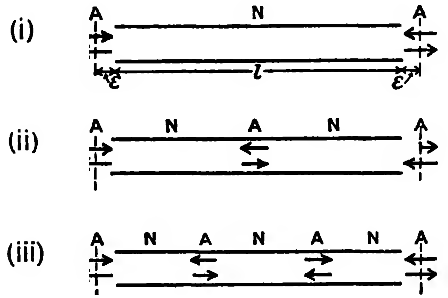


FIG. 463

instant, and the lower arrows show the motion half a period later. The wave-length (λ_0) is equal to $2(l+2\epsilon)$, so that the frequency (n_0) is given by

$$n_0 = \frac{c}{\lambda_0}$$

$$= \frac{c}{2(l+2\epsilon)}$$

$$= \frac{1}{2(l+2\epsilon)}\sqrt{\frac{\gamma p}{\rho}}$$

The first overtone is shown in Fig. 463 (ii). The wave-length is $(l+2\epsilon)$, so that the frequency (n_1) is given by

$$n_1 = \frac{c}{l+2\epsilon}$$

$$= 2n_0$$

The first overtone is, therefore, an octave above the fundamental. The second overtone (Fig. 463 (iii)) has a wave-length of $\frac{2}{3}(l+2\epsilon)$, so that its frequency (n_2) is given by

$$n_2 = \frac{c}{\frac{2}{3}(l+2\epsilon)}$$

$$= \frac{3}{2}n_1$$

$$= 3n_0$$

Thus the relationship between the fundamental and the overtones is the same as in the case of the stretched string, the frequencies being proportional to the numbers 1, 2, 3, 4, etc. and thus forming a harmonic series. In fact the stretched string and the pipe open at both ends are

exactly similar except for the fact that nodes in the former case are replaced by antinodes in the latter and *vice versa*.

Tube Closed at One End and Open at the Other.—In Fig. 464 (i) a longitudinal disturbance (represented as a transverse one) is travelling down a tube closed at P and open at Q. The displacement is reversed on reflection at the fixed end P (page 593), so that, whereas in the incident disturbance all particles were displaced to the right of their mean position, they are now displaced to the left. Consequently the rarefaction in the front part of the incident disturbance remains a rarefaction after reflection (Fig. 464 (ii)). At the open end Q the reflection is of the opposite type but, as explained on page 648, it must be regarded as occurring a short distance beyond the end. In (iii) the reflection has taken place as described on page 649. In order to complete the cycle begun in (i) it is necessary for the disturbance to be reflected at the closed end a second time so as to restore it to its original character (Fig. 464 (iv)), and then again at the open end, after which it reaches its initial position and state of motion (v).

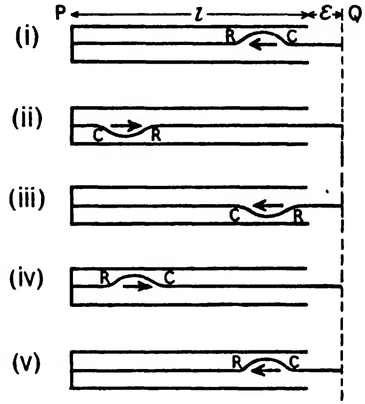


FIG. 464

The passage of the disturbances backwards and forwards along the tube can be represented by two progressive waves as in Fig. 465. The wave-length of each wave is equal to four times the distance between the two

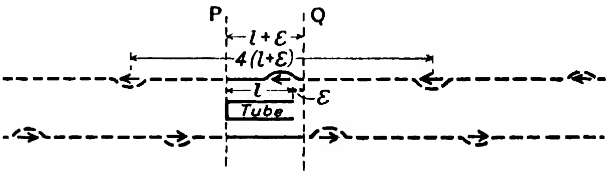


FIG. 465

planes where reflection occurs, *i.e.* $4(l+\epsilon)$, where l is the actual length of the tube. If the original disturbance is not a pure sine wave, the progressive waves will be a series of pairs of sine waves of appropriate amplitude and wave-length. Each pair of these Fourier components will produce a stationary wave. Any sine wave is a possible component of the vibration provided that its wave-length is such as to produce a node at the closed end of the tube and an antinode at a distance ϵ beyond the

open end. The possible modes of vibration of the air column are, therefore, as follows.

The fundamental mode is shown in Fig. 466 (i). The wave-length (λ_0) is equal to $4(l+\epsilon)$, so that the frequency (n_0) is given by

$$n_0 = \frac{c}{\lambda_0}$$

$$= \frac{c}{4(l+\epsilon)}$$

$$= \frac{1}{4(l+\epsilon)}\sqrt{\frac{\gamma p}{\rho}}$$

It will be noticed that this fundamental frequency is the same as that of a tube of length $2l$ which is open at both ends provided that ϵ is the same in each case. This fact is obvious even without comparing the two expressions for the fundamental frequencies, because two vibrating columns such as that in Fig. 466 (i), when placed end to end with their nodes together, will give a system equivalent to that shown in Fig. 463 (i). If we compare the fundamental modes of two tubes of *equal* length l, one with a closed end and an open end and the other with two open ends, we see that the wave-lengths are not exactly in the ratio 2 : 1. The former is $4(l+\epsilon)$ and the latter is $2(l+2\epsilon)$. Thus if the closed end of a tube is opened, it is not true to say that its fundamental frequency is doubled unless the end corrections are very small. The first overtone of a tube closed at one end is shown in Fig. 466 (ii). The wave-length is $\frac{4}{3}(l+\epsilon)$, so that the frequency n_1 is given by

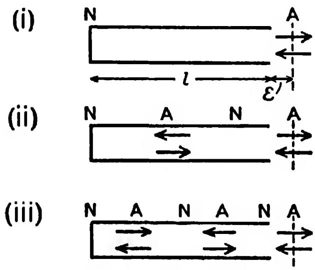


FIG. 466

$$n_1 = \frac{c}{\frac{4}{3}(l+\epsilon)}$$

$$= 3n_0$$

The second overtone has a wave-length of $\frac{4}{5}(l+\epsilon)$ and a frequency $5n_0$. Thus the frequencies of the fundamental and overtones are proportional to the odd numbers 1, 3, 5, etc. The 2nd, 4th, 6th . . . members of the harmonic series are missing.

4. THE RESONANCE OF AIR COLUMNS

In the preceding section of this chapter we discussed the natural vibrations of columns of air, and have deduced the frequencies of the various possible modes of vibration of open and closed tubes. We now

consider the excitation of the air in a tube by an external source of sound of definite frequency.

Suppose that a source of sound whose frequency can be varied is placed near the end of a column of air contained in a tube. A telephone earpiece connected to a valve oscillator would be a very suitable source. As the frequency of the source is gradually increased from zero, a stage is reached at which the sound becomes very much louder. This occurs because the frequency of the source has become equal to the fundamental frequency of the air column, which is therefore set into resonant vibration. As the frequency of the source is raised above this value the intensity of the sound again falls, but with a further continuous increase of frequency the various overtones of the tube are heard to resound in turn. It should be noted that by judging the condition of resonance by loudness we are dealing with energy resonance and not amplitude resonance (page 613), because loudness depends on the energy emitted by a sounding body. When resonance occurs, therefore, the natural undamped frequency of the air column is exactly equal to the frequency of the source. When the frequency of the source is such that the air column is not resonating, the air is nevertheless performing forced vibrations (page 607) whose amplitude will, in general, be much smaller than that of the resonant vibrations. The resonance of air columns can also be investigated by using a source of fixed frequency and a tube of adjustable length.

Closed Resonance Tube.—The term "closed tube" is used to denote a uniform tube open at one end and closed at the other. In a closed resonance tube the length of the enclosed air is made adjustable, either by making the tube of two parts which telescope into each other, or by fitting a sliding piston to act as a movable closed end. Alternatively, the tube may be held vertically and raised or lowered in a wider tube containing water, the water surface acting as the closed end. There are also other ways in which water may be used to provide the necessary adjustment of length. A tuning-fork is the most commonly used source of sound in connection with a resonance tube in an elementary laboratory. The fork is struck and held near the open end of the tube while the length of the air column is gradually increased from its smallest possible value. Eventually a resonance is obtained and the length of the column is adjusted as nearly as possible to give the maximum response. Since we have found by this method the *shortest* air column which resonates to the fork, we know that the column is vibrating in its fundamental mode. This state of affairs is shown in Fig. 467 (i), in which the closed end of the tube is represented as a movable piston. The length of the air column (l_1) is measured from the open end of the tube to the closed end. The air column is then lengthened by withdrawing the piston while the fork is sounding. The second position of resonance (Fig. 467 (ii)) is reached when the length of the air column is approximately $3l_1$. Its length (l_2) is measured.

The wave-length (λ) in air of the sound emitted by the fork can be deduced from these observations. It will be noticed that the unknown end correction ϵ prevents us from finding λ from a single observation of resonance, but by taking two resonances ϵ can be eliminated. Thus in the first case

$$\frac{\lambda}{4}=l_1+\epsilon$$

and in the second

$$\frac{3\lambda}{4}=l_2+\epsilon$$

Therefore by subtraction

$$\frac{\lambda}{2}=l_2-l_1$$

This relationship should be obvious from inspection of Figs. 467 (i) and (ii). The wave-length having been determined as $2(l_2-l_1)$, we can use the

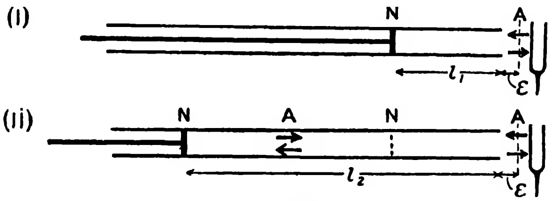


FIG. 467

relationship $c=n\lambda$ to determine c if n is known or n if c is known. Again, if a fork of known frequency n gives tube lengths of l_1 and l_2 for successive resonances and another fork of unknown frequency n' gives corresponding lengths l_1' and l_2', we have

$$2(l_2-l_1)=\frac{c}{n}$$

and

$$2(l_2'-l_1')=\frac{c}{n'}$$

so that

$$\frac{n'}{n}=\frac{l_2-l_1}{l_2'-l_1'}$$

and n' may be calculated.

Further resonances beyond the second can, of course, be obtained if the tube is long enough. The higher the frequency of the fork, the shorter the wave-length and the greater the number of resonances which can be obtained with a tube of given length. The distance through which the piston is moved between consecutive resonances is always half a wave-length.

The value of ϵ can easily be obtained from measurements of l_1 and l_2. Thus $(l_2 - l_1)$ is equal to $\lambda/2$, so that $\lambda/4$ is equal to $\frac{1}{2}(l_2 - l_1)$. But $\lambda/4$ is the distance from the antinode outside the tube to the closed end of the tube in Fig. 467 (i), *i.e.* it is equal to $l_1 + \epsilon$. Therefore

$$l_1 + \epsilon = \tfrac{1}{2}(l_2 - l_1)$$

so that

$$\epsilon = \tfrac{1}{2}(l_2 - 3l_1)$$

As an example to indicate the order of magnitude of the lengths l_1 and l_2 in a particular case, we can consider a fork giving the octave above middle C and therefore having a frequency of 512 c.p.s. If we assume the speed of sound in air at room temperature to be 33,400 cm. sec.$^{-1}$, the wave-length of this sound in air is (33,400 ÷ 512) or 65·2 cm. Thus the increase of length of the tube between consecutive resonances ($\lambda/2$) is equal to 32·6 cm. in this case, and $\lambda/4$ is 16·3 cm. For a tube of roughly 3 cm. diameter the value of ϵ is about 1·0 cm., so that

$$\begin{aligned} l_1 &= \frac{\lambda}{4} - \epsilon \\ &= (16{\cdot}3 - 1{\cdot}0)\text{ cm.} \\ &= 15{\cdot}3\text{ cm.} \end{aligned}$$

and

$$\begin{aligned} l_2 &= l_1 + \frac{\lambda}{2} \\ &= (15{\cdot}3 + 32{\cdot}6)\text{ cm.} \\ &= 47{\cdot}9\text{ cm.} \end{aligned}$$

When a resonance tube is set into oscillation by holding a tuning-fork near it or by blowing across its open end, the sound which the tube emits dies away very rapidly when the driving agent is suddenly stopped. This indicates that the oscillations of the air column are highly damped, and it accounts for the fact that the resonances are not very sharp (page 613). That is to say, a considerable alteration in the length of the tube (say 2 or 3 mm. in 15 cm.) has very little effect on the loudness of the response. For this reason it is important that several independent settings should be made for each resonance position and a mean value deduced for the length of the air column in each case. Accuracy of determination of the resonant lengths is particularly important in the experiment described in the next paragraph.

Suppose that several tuning-forks of known frequency are available and each is used in turn with a given resonance tube. Let l be the

shortest length of the tube which resonates to a fork of frequency n. The wave-length is c/n, so that

$$4(l+\epsilon)=\frac{c}{n}$$

or

$$l=\frac{c}{4n}-\epsilon$$

Thus if the value of l is plotted against $1/n$ for each fork, a straight line should be obtained. The slope of the line is the coefficient of $1/n$, *i.e.* $c/4$, and the intercept on the l axis is $-\epsilon$.

Example.—*A resonance tube has a jagged end and it is used to find the velocity of sound in air. A tuning-fork of frequency* 250 *causes it to resound when it is filled with water to a mark* 28 *cm. below a reference mark near the open jagged end. A fork of frequency* 500 *causes resonance when the water reaches a mark* $11\frac{1}{2}$ *cm. below the reference mark. Both these resounding lengths are the shortest possible with these forks. Find the velocity of sound from these results, and also the position of the antinode at the upper end in relation to the mark.* (L.H.S.)

Let x cm. be the distance of the antinode above the reference mark. Then, when the air column is resounding to the 250 c.p.s. fork, its effective length is $(28+x)$ cm. In the case of the 500 c.p.s. fork the effective length is $(11{\cdot}5+x)$ cm. Each of these effective lengths is one-quarter of the wave-length, and if c is the velocity of sound in air we have

$$c=250\times4(28+x)$$

and

$$c=500\times4(11{\cdot}5+x)$$

Therefore

$$250\times4(28+x)=500\times4(11{\cdot}5+x)$$

$$28+x=2(11{\cdot}5+x)$$
$$=23+2x$$

so that

$$x=28-23$$
$$=5 \text{ cm.}$$

Substituting this value for x in, say, the first of the above expressions for c gives

$$c=250\times4(28+5)$$
$$=33{,}000 \text{ cm. sec.}^{-1}$$

Open Resonance Tube.—A resonance tube which is open at both ends is less commonly used than the closed tube, but it is capable of providing an instructive experiment. It can be constructed of thin-walled tube, *e.g.* brass or glass, and its various parts must be telescopic. If more than one position of resonance is to be investigated with any given fork it is necessary for the length of the tube to be at least doubled during the experiment, and extensions made of stiff paper are very useful for this purpose.

Figs. 468 (i) and (ii) show respectively the state of vibration of the open tube for the first and second resonances. In (i) the tube is vibrating in its fundamental mode, and in (ii) it is sounding its first overtone. The wave-length (λ), being determined by the frequency of the fork, is the same in both cases. In (i) we have

$$\frac{\lambda}{2}=l_1+2\epsilon$$

and in (ii)

$$\lambda=l_2+2\epsilon$$

Therefore, by subtraction,

$$\frac{\lambda}{2}=l_2-l_1$$

Thus λ can be determined and either n or c calculated from the relationship $c=n\lambda$.

If several forks of known frequency are available and the shortest

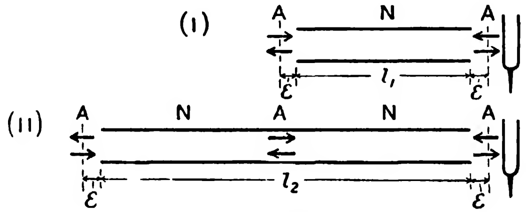


Fig. 468

length (l) of the open tube which will resonate is found for each fork in turn, a straight-line graph may be drawn between l and the reciprocal of the frequency of the fork, just as with the closed tube. The appropriate equation is

$$l=\frac{c}{2n}-2\epsilon$$

Kundt's Tube.—If dry light dust, such as cork dust or lycopodium, is distributed uniformly along the bottom of a horizontal glass tube, and the air in the tube is then caused to vibrate strongly in one of its higher modes, the dust will arrange itself into a pattern which indicates the positions of the nodes and antinodes. A solid rod vibrating longitudinally is often used as the source of sound in the elementary version of this experiment. This source is chosen because it provides a high-pitched note of short wave-length in air and its energy is sufficient to disturb the dust. Moreover, the experiment then provides a comparison between the speed of sound in the solid and in air.

The apparatus is shown diagrammatically in Fig. 469. The rod is clamped at its midpoint, and one of its free ends, which carries a light

disc Q, protrudes into the glass tube. The disc is smaller in diameter than the tube and not in contact with the walls. The other end of the tube is closed with a bung. In order to allow the dust to move freely under the action of the vibrating air it is essential that both it and the inside wall of the tube should be clean and dry. The rod can be set into longitudinal vibration by grasping it with a resined cloth near the centre and pulling towards the free end. Its longitudinal vibrations will be mainly in the fundamental mode, *i.e.* with an antinode at each free end and a single node in the middle (page 645). The wave-length of the stationary wave in the solid rod is therefore equal to $2L$, where L is the length of the rod.

While the rod is being made to sound, the tube is moved to and fro along its axis, thereby adjusting the length of the air column between the vibrating disc at Q (Fig. 469) and the closed end of the tube, until the dust patterns are seen to be at their best. One of the modes of vibration of the air column is then in resonance with the rod. This resonance

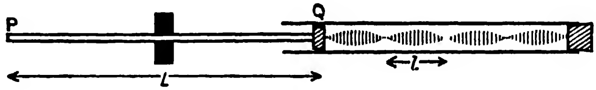


FIG. 469

cannot usually be judged by ear, because the sound emitted by the tube is weak in comparison with that of the rod. The patterns are often tolerably well formed even without adjustment of the air column. This adjustment has no effect on the wave-length, of course, since this is fixed by the vibration of the rod.

The dust patterns are of the type indicated in Fig. 469. Small regularly spaced regions of undisturbed dust which locate the nodes (because there is no vibration of the air at a node) are separated by regions of disturbance whose intensity is greatest near their centres where the antinodes are situated. The disturbance takes the form of transverse striations or ridges, the cause of which we need not discuss. The stopped end of the tube is a node, and the disc attached to the rod is an approximate node because its amplitude is very much smaller than that of the air at the antinodes.

When satisfactory dust figures have been obtained, the mean distance between consecutive nodes can be deduced from measurements of the pattern, and this is equal to half the wave-length of the sound in air. If this distance is l and the frequency of the rod is n, then the speed of sound (c_g) in the gas in the tube is equal to $2nl$, while the speed of longitudinal waves in the solid of the rod (c_s) is $2nL$, since L is the distance between two antinodes. Therefore

$$\frac{c_s}{c_g} = \frac{L}{l}$$

The experiment therefore enables us to determine c_s/c_g, and this information can be used in a variety of ways. For example, since the speed of sound in air is known, the value of c_s for the material of the rod can be determined. On page 572 it is stated that

$$c_s = \frac{\quad}{\rho_s}$$

where q is Young's modulus for the solid and ρ_s is its density. Thus q can be calculated if ρ_s is determined independently. It may be mentioned here that a determination of q may be made acoustically without the use of the Kundt's tube experiment provided that the frequency of the note emitted by the rod is determined by some means or other. One way of doing this is to tune a sonometer wire to the note and then to alter the length of the wire until it is in unison with a fork of known frequency. The frequency of the rod (n) is then found by inverse proportion and c_s is calculated from the equation $c_s = 2nL$. This experiment is not usually

FIG. 470

very accurate on account of the difficulty of tuning the sonometer wire to the high-pitched note of the rod.

If c_s for the material of the rod is known from a Kundt's tube experiment in which the value of the speed of sound in air is assumed, the same rod can be used in a similar experiment with another gas replacing the air in the tube. The speed of sound in the gas is then found. Alternatively, as was done by Kundt, two tubes may be used simultaneously, one containing air and the other the gas in which the speed of sound is to be measured. The rod has a disc on each end and is thus able to set up vibrations in both tubes at once. From measurements of the patterns formed by the dust in each tube a direct comparison of the speed of sound in a gas can be made with that in air. A further elaboration of the experiment is shown in Fig. 470. The gas is contained in a long glass tube A, closed at both ends and containing dust. One end of the tube projects into another tube B containing air and dust and fitted with a movable piston at its far end. The glass of tube A is set into longitudinal vibration by stroking and, when the correct tuning is obtained, dust figures are formed in both A and B.

It has been shown (page 573) that the speed of sound in a gas is equal to

$$\sqrt{\frac{\gamma p}{\rho}}$$

where γ is the ratio of the specific heat of the gas at constant pressure to the specific heat at constant volume, p is the pressure of the gas and ρ is

its density under prevailing conditions. It follows that any experiment for the determination of c_0 such as those just described can be used to yield a value for γ for the particular gas used. In Vol. 2 of this book (page 433) it is explained how the value for any particular gas depends upon the number of degrees of freedom of its molecules which, in turn, is fixed by the number of atoms present in each molecule. This information can therefore be derived from acoustical experiments, which provide some of the most accurate data for γ.

In a more modern form of the Kundt's tube experiment the source of sound is a diaphragm of magnetic alloy placed across the end of the tube. This is made to vibrate by means of an electromagnet fed with an alternating current from a valve oscillator. A telephone earpiece or loudspeaker unit can be adapted for the purpose. Alteration of the length of the air column is effected by a movable piston at the other end of the tube. In this form of the experiment a greater amount of energy can be communicated to the gas column, with the result that its various resonances can be distinguished by ear, and what is more important, the dust is so violently disturbed at the antinodes that it becomes suspended right across the cross-section of the tube at these places, forming thin "antinodal discs" as they are called. On account of the steadiness and sharpness of the discs their positions can be determined much more accurately than those of the nodal heaps, and the accuracy of the wave-length determination is greatly enhanced.

5. ORGAN PIPES

A pipe organ contains a number of sets of pipes, each set consisting of a range of pipes of similar musical quality whose pitch varies from one pipe to the next according to the equally tempered scale. Each range of pipes can be connected in turn or in combination to the keyboard by withdrawing the appropriate stop or stops. Thus when several stops are withdrawn the operation of a single key on the keyboard causes pipes of different quality to speak simultaneously, their pitches usually being related by octaves. There is a big variation in musical quality from one stop to another, *e.g.* between flute, clarinette and trumpet (all of which are intended to imitate the corresponding orchestral instruments). Other stops are intended to imitate string tone, but the most characteristic tone of the organ is provided by the diapason stops.

The acoustic principle embodied in the organ pipe and in wind instruments in general is the resonance of the air in the pipe to a source of sound situated at one end and operated by an air stream. In the case of the flute and flue organ pipes, the frequency emitted is controlled to a much larger extent by the resonating air column than by the source, because the former has by far the greater amount of energy, so that if any inequality of frequency does exist between the source and the air column it is the source which is pulled into line and not *vice versa*.

In the brass instruments of the orchestra the source of sound is the lips of the player which, when pressed against the mouthpiece, are caused to vibrate by the passage of air between them. The natural frequency of the air column is varied by altering its length, *e.g.* by means of piston valves in the horn and the slide in the trombone. In instruments such as the flute, clarinette or bassoon, the frequency of the air column is varied by the opening or stopping of holes in the side of the tube. The relative sizes and positions of these holes have evolved empirically through years of instrument-making, and the theory of such instruments is difficult.

From the point of view of the method by which the sound is produced, organ pipes can be divided into two classes—flue pipes and reed pipes.

The Flue Pipe.—A flue organ pipe is frequently made of wood, in which case it has a uniform square cross-section. Otherwise it is made of metal and is cylindrical in shape. Pipes of the latter type are often used for ornamental purposes on the fronts of organs. The characteristic of the flue pipe is that **edge tones** are responsible for the initiation of the sound. If air is forced through a narrow straight slit and a solid wedge is placed symmetrically in the path of the jet with its edge parallel to the slit, a musical note is produced which is called an "edge tone." An authoritative account of the mechanism of edge tones has been given by Dr. G. Burniston Brown, who introduced smoke into the air jet in order to make its movements visible and then examined it by light which was interrupted with the same frequency as the note emitted.

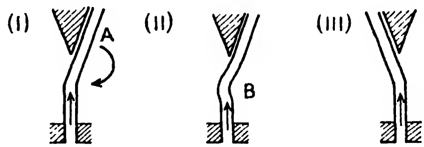


FIG. 471

The essence of Dr. Burniston Brown's explanation of the mechanism of edge tones is as follows. If the wedge is symmetrically placed and the jet suffers no disturbance from external causes, the air stream will be cut in half by the wedge. A slight disturbance of the jet will, however, send it to one side or the other of the wedge, *e.g.* to the right as in Fig. 471 (i). This causes a momentary compression to be formed in the air outside the jet in the region marked A. This compression spreads out from A in all directions on the right-hand side of the jet and, in particular, it reaches B (Fig. 471 (ii)), where it has the effect of pushing the oncoming jet over to the left. This causes the jet to go to the left of the wedge (Fig. 471 (iii)), so that compression is produced on the left and the jet is sent back. The cycle of events is repeated indefinitely when conditions are favourable, and the compressions are propagated into the surrounding air, thus causing a sound to be heard.

It is not necessary for us to discuss how the frequency of the edge tone depends upon the speed of the jet and the distance from the slit to the wedge, but it may be mentioned that as this distance is increased from

the minimum distance for which a tone is produced, the pitch falls steadily at first and then suddenly rises. After this there is a further steady fall in pitch, followed by an abrupt rise and so on. A similar discontinuous change of pitch is observed when the speed of the jet is gradually reduced while the wedge is kept stationary.

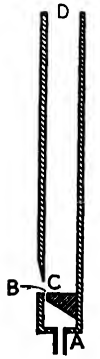


FIG. 472

The Open Flue Pipe.—Fig. 472 is a diagram of a wooden flue pipe of the open diapason type. When the appropriate key on the organ keyboard is depressed, a mechanical, pneumatic or electrical mechanism opens a tube which connects A to the wind chest. The air stream therefore enters the small box at the bottom of the pipe and issues through the straight slit B which is called the flue. The stream of air then impinges on the sharp edge of the lip C and edge tones are produced. The portion of the pipe CD encloses an air column and behaves as a pipe with two open ends. If the frequency of the edge tone is exactly, or even only approximately, equal to the frequency of one of the longitudinal modes of vibration of the air column, this vibration is set up. As already mentioned, the energy of the edge tone is very much smaller than that of the vibrating column, so that the only possible notes of any appreciable intensity which the pipe emits are the fundamental and overtones of the air column. It is the aim of the organ builder to adjust the position of the lip and the rate of flow of air into the tube so as to cause the pipe to sound its fundamental strongly. Overtones are always present, however.

The end correction, *i.e.* the distance of the antinode outside the open end of the pipe, is of comparable magnitude with that appropriate to a cylindrical tube, *i.e.* about one-third of the transverse dimension of the pipe. At the lip end of the air column, however, the movement of the air is more hampered than at the top of the tube, so that this region is even further from what would be the position of the antinode if the stationary wave were continued beyond the lip end. The end correction here is greater, therefore, being perhaps one and a half times the transverse dimension of the pipe. If we call this correction ϵ_1, and the smaller correction at the top ϵ_2, the effective length of an open organ pipe of actual length l is $(l+\epsilon_1+\epsilon_2)$. In the fundamental mode of vibration the two antinodes situated at this distance apart have only one node midway between them (Fig. 473 (i)). The wave-length is therefore $2(l+\epsilon_1+\epsilon_2)$, and the frequency of the fundamental (n_0) is given by

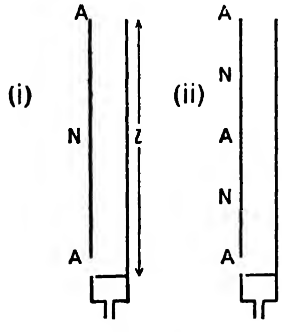


FIG. 473

$$n_0=\frac{c}{2(l+\epsilon_1+\epsilon_2)}$$

where c is the speed of sound in the air in the pipe. In the next mode (the first overtone) shown in Fig. 473 (ii) the wave-length is $(l+\epsilon_1+\epsilon_2)$, and the frequency (n_1) is given by

$$n_1 = \quad l+\epsilon_1+\epsilon_2$$

$$= 2n_0$$

The next overtone has a frequency $3n_0$, the next $4n_0$ and so on, exactly as described on page 650, where the modes of vibration of an open pipe have already been discussed. It must be mentioned, however, that in fact the corrections ϵ_1 and ϵ_2 depend on the frequency. This causes the frequencies of the overtones to differ from whole multiples of the fundamental frequency, so that, in practice, the fundamental and overtones do not exactly constitute a harmonic series.

It must not be supposed that the energy of the fundamental tone of an organ pipe, or indeed of any musical instrument, is very much greater than that of the individual overtones. It is true that in the open diapason the energy of the overtones falls off fairly rapidly with increasing frequency, but in the trumpet stop the energy of each overtone up to about the sixth is actually greater than that of the fundamental.

If a single pipe is removed from an organ and its performance investigated experimentally, it will be found that the sound emitted depends very much upon the speed with which the air is blown into it. This is due to the dependence of the frequency of the edge tones on wind speed. If the pressure of blowing is gradually increased from the correct value for the normal performance of the pipe, the sound emitted does not change immediately, however, because the air column in the pipe holds the frequency constant even though, without the controlling influence of the air column, the frequency of the edge tone appropriate to the enhanced pressure has been raised. Eventually there is a jump, however, and the first overtone comes in strongly, the fundamental subsequently disappearing altogether. The pipe is then said to be "overblown." Further increase of pressure eliminates the first overtone as well as the fundamental. It is also possible to elicit the first overtone without the fundamental by *under-blowing* the pipe.

The tuning of an open organ pipe is done by making a change in its effective length. In some cases a slot in the wall of the pipe near the top end can be opened and closed by a wooden slide. Opening the slot decreases the effective length of the pipe and so raises its pitch. In other cases the actual length of the pipe can be altered by means of a sleeve which fits over the top. Any obstacle placed near the open end of an organ pipe hinders the free movement of the air and sends the antinode further from the end, thereby increasing the effective length of the pipe and so lowering its pitch.

The vibration of the air column in an organ pipe can be demonstrated

in a variety of ways. One of the most direct methods is to introduce light dust or smoke particles into the pipe and to observe or photograph them through a glass window. The particles take up the movement of the air and the path of each one is seen as a short straight line. The lines have their smallest length at the nodes and their largest at the antinodes.

Another method involves the use of a hot-wire probe, the action of which has already been mentioned on page 575. It consists of a fine platinum wire through which an electric current is passing. When this is held in the air column at right angles to the direction of the longitudinal vibrations, it is cooled by the alternating flow of air past it. The resulting decrease in the resistance of the wire can be measured electrically, and provides a measure of the particle velocity of the air in its neighbourhood if the apparatus has previously been calibrated by placing it in a unidirectional air stream and measuring its resistance at various known air speeds.

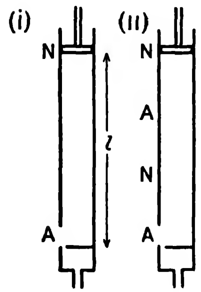


FIG. 474

In a device called a **manometric capsule** for observing the oscillations of the air in an organ pipe, a hole is made in the wall of the pipe and is covered with a thin rubber membrane. This diaphragm forms the back of a small chamber which is fixed on the outside of the pipe wall. Gas from the main supply passes through this chamber on its way to a small burner. When the pipe is sounding, the pressure variations in the air column near the diaphragm cause the latter to vibrate, with the result that corresponding fluctuations of pressure are superposed on the gas stream which reaches the burner. The flame therefore rises and falls with the frequency of the vibrations of the air in the pipe in the neighbourhood of the membrane. The variations in the height of the flame will, in general, be too rapid to see, but if the flame is viewed in a vertical mirror which is rotating about a vertical axis, the image is drawn out into a horizontal band of light, the top edge of which is seen to be serrated when the pipe is sounding. If the pipe is overblown so that the first overtone sounds strongly, the number of serrations is doubled. The manometric capsule is operated by the *pressure* fluctuations in the pipe, and therefore it will not indicate the vibrations if it is placed at an antinode of the stationary wave. In order to render the flame luminous, the gas may be passed through a tube containing filter-paper soaked in benzene.

The Stopped Flue Pipe.—A stopped pipe of the diapason type is similar to an open diapason pipe, except that its upper end is closed with a well-fitting plunger which can be moved in and out to vary the length of the air column (Fig. 474). The effective length of the column is $l+\epsilon_1$, there being a correction ϵ_1 at the lip end and no correction at the closed end. Any mode of vibration is possible which will give a node at the closed end and an antinode at the lip end (page 652). The

fundmental mode is therefore as shown in Fig. 474 (i). In this the wave-length is $4(l+\epsilon_1)$, and the frequency n_0 is given by

$$n_0=\frac{c}{4(l+\epsilon_1)}$$

The first overtone is shown in Fig. 474 (ii). The wave-length is $\frac{4}{3}(l+\epsilon_1)$, the frequency $\frac{3c}{4(l+\epsilon_1)}$, which is $3n_0$. Similarly, the next overtone has a frequency of $5n_0$ and so on. The frequencies of the fundamental and overtones are therefore proportional to the odd numbers 1, 3, 5 . . . Thus, whereas the full harmonic series of tones can be present in an open pipe, only the odd members of the series can occur in a stopped pipe.

If the end corrections are ignored, the expression for the fundamental frequency of an open pipe becomes $\frac{c}{2l}$ and that for a stopped pipe of the same length $\frac{c}{4l}$. Thus, to the degree of approximation involved in ignoring the end corrections, we can say that the pitch of a closed pipe is an octave below that of an open pipe of the same length.

In quantitative and numerical exercises on organ pipes it is frequently intended that end corrections shall be ignored—often it is not possible to solve the problem without ignoring them, since their values must not be assumed.

Reed Pipes.—The construction of a reed organ pipe is shown diagrammatically in Fig. 475. The reed A is a tongue of brass fixed at its upper end to the outside of a box B known as a "shallot." When air is sent into the "boot" (C) of the pipe it streams into the shallot through the opening D, at the same time causing the reed to vibrate into and out of the opening, thereby interrupting the stream at a rate equal to the natural frequency of the reed. The air vibrations are more energetic and of a more definite frequency than the edge tones of a flue pipe, so that it is necessary to tune them to the correct frequency required from the pipe. This is done by altering the effective length of the reed by moving the tuning wire E up or down. The length of the pipe must be adjusted so that it resonates with the reed. It should be mentioned that the pipe is effectively closed by the reed so that its lower end is approximately a node.

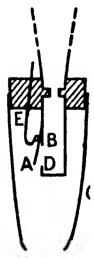


FIG. 475

Dependence of the Pitch of an Organ Pipe on Temperature.—It is evident from the formulæ for the fundamental frequencies of open and stopped flue pipes that for a given pipe this frequency is proportional to c, the speed of sound in the gas in the pipe. Thus, since c is proportional to the square root of the absolute temperature, so also is the fundamental frequency.

***Example.**—If the frequency of an organ pipe is* 64·0 *c.p.s. when it is blown with air at* 18° *C., calculate its frequency when blown with* (a) *air at* 24° *C. and* (b) *hydrogen at* 18° *C.* (*The relative density of air to hydrogen* = 14·4.) (L.I.)

(*a*) Since the speed of sound in a gas is proportional to the square root of the absolute temperature, and the frequency of a given organ pipe is proportional to the speed of sound in the gas with which it is blown (the wave-length being fixed by the length of the pipe), we can write

$$\frac{\text{Frequency at } 24^\circ \text{ C.}}{\text{Frequency at } 18^\circ \text{ C.}} = \sqrt{\frac{273+24}{273+18}} = \sqrt{\frac{297}{291}}$$

Therefore the frequency of the pipe when blown with air at 24° C. is equal to

$$64{\cdot}0 \times \sqrt{\frac{297}{291}}$$

$$= 64{\cdot}7 \text{ c.p.s.}$$

(*b*) Let ρ be the density of hydrogen at 18° C. and the prevailing pressure (p). Then the density of air at the same temperature and pressure is $14{\cdot}4\rho$. We have therefore

$$\text{Speed of sound in hydrogen} = \sqrt{\frac{\gamma p}{\rho}}$$

and

$$\text{Speed of sound in air} = \sqrt{\frac{\gamma p}{14{\cdot}4\rho}}$$

The value of γ is the same for both air and hydrogen. Since the wave-length is the same in each case, we have

$$\frac{\text{Frequency when blown with hydrogen at } 18^\circ \text{ C.}}{\text{Frequency when blown with air at } 18^\circ \text{ C.}} = \sqrt{14{\cdot}4}$$

so that

$$\text{Frequency when blown with hydrogen at } 18^\circ \text{ C.} = 64{\cdot}0\sqrt{14{\cdot}4}$$
$$= 242{\cdot}9 \text{ c.p.s.}$$

6. ULTRASONICS

It has already been mentioned that the human ear cannot hear vibrations of a frequency exceeding about 20,000 c.p.s. however intense they may be. High frequency vibrations above this upper limit of audibility are called **ultrasonic.** They have a wave-length in air of less than about 1·6 cm.

The Production of Ultrasonic Vibrations.—It is quite possible to make a whistle and even a tuning-fork of such small dimensions as to produce ultrasonic vibrations, but these methods are not commonly used.

One of the commonest types of source of ultrasonics depends on the **piezo-electric effect** which is exhibited by certain crystals, *e.g.* quartz and Rochelle salt. Natural quartz often occurs in crystals consisting of hexagonal prisms with hexagonal pyramids at each end. Fig. 476 (i) is a drawing of the hexagonal-prism part of a quartz crystal, and certain directions in the crystal are important in connection with the effect which

is about to be described. These are the optic axis marked O in Fig. 476 (i), which is any direction parallel to the axis of the hexagonal prism, three electric axes which are directions parallel to the lines E passing through opposite corners of the hexagonal cross-section, and three mechanical axes (M) which are directions perpendicular to the sides of the hexagon. In order to make use of the piezo-electric effect, a rectangular block is cut from the crystal as shown by the shading so that its three sets of edges are parallel respectively to the optic axis, one of the electric axes and one of the mechanical axes. An end view of the block is shown in Fig. 476 (ii), the optic axis being perpendicular to the plane of the paper. Two metal plates are placed in contact with the faces which are perpendicular to the electric axis. If the block is stretched along the mechanical axis or compressed along the electrical axis, equal and opposite electric charges are produced on these plates, their magnitude being proportional to the

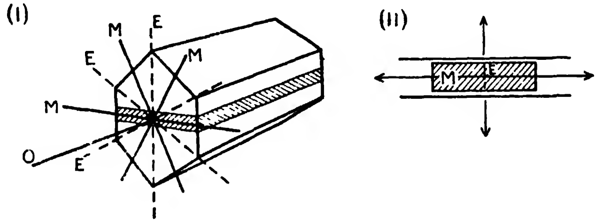


FIG. 476

mechanical strain. If the direction of the mechanical strain is reversed the signs of the charges are reversed. The converse effect also exists; that is to say, if an electrical potential difference is applied to the plates, then the block contracts along E and elongates along M, or *vice versa*, according to the direction of the potential difference. If the potential difference is alternating, *i.e.* changing its sign with a definite frequency, the quartz block is put into a state of longitudinal forced vibration, along the directions E and M, and when the frequency of the alternating potential difference is made equal to the frequency of one of its natural modes of vibration, comparatively large resonant vibrations are set up. The fundamental modes of longitudinal vibration consist of antinodes at opposite faces and a single node midway between, as in the longitudinal vibrations of a rod. Thus the wave-length (in the quartz) of the fundamental vibration is equal to double the linear dimension of the block (say l) in the direction of the vibration, and if c is the speed of the ultrasonic waves in the quartz in this direction, the frequency is $c/2l$. Owing to its crystalline nature, quartz is not isotropic and c is different in different directions.

In order to produce the necessary alternating potential difference a valve oscillator is used, within the circuit of which the quartz block between the two conducting plates forms an electrical condenser. The

tuning of the circuit to the frequency of the natural vibration of the quartz which it is desired to excite is done by means of another variable condenser. The resonance is very sharp and the natural frequency of the quartz is very constant if its temperature is maintained constant, so that the frequency of the valve oscillator is stabilized by the inclusion of the quartz crystal in its circuit. Such an arrangement is the basis of the quartz block. Evidently a quartz block situated in air or a liquid can be made to generate longitudinal ultrasonic waves in the surrounding fluid, the direction of emission from the quartz being either up and down or left and right in Fig. 476 (ii), according to which mode of vibration is excited by the particular frequency of the applied alternating potential difference. The metal-plate electrodes may be replaced by a film of metal deposited on the two opposite faces when it is intended that waves should be emitted from these faces. Overtones are often made use of in order to obtain higher frequencies than the fundamental modes.

When a rod of ferromagnetic (*i.e.* strongly magnetizable) metal or alloy is magnetized its length changes slightly, the amount of the change depending upon the nature of the material and the strength of the magnetizing field. This phenomenon is known as **magnetostriction.** If insulated wire is wound round the rod of magnetic material and an alternating electric current is passed through the coil, then the magnetizing field alternates in step with the current, reversing its direction every time the current does so. The magnetostrictive effect is not reversed when the field is reversed, however, so it is usual to pass a steady current through another coil wound on the rod so as to produce a steady magnetic field of greater strength than the alternating one. The combined effect of the steady and alternating fields is to make the intensity of magnetization of the rod fluctuate with the same frequency as the alternating current above and below the intensity produced by the steady current. The rod therefore lengthens and shortens in step with the alternating current, and resonance will occur if the frequency of the current is made equal to that of the fundamental mode of longitudinal vibration of the rod (an antinode at each end and a node in the centre) or to any of the overtones.

Determination of Ultrasonic Wave-Length.—A Kundt's-tube arrangement may be used to measure the wave-length of ultrasonic waves in air or gases. The ultrasonic source is placed at one end of the glass tube and the other end is closed by a movable piston. Smoke particles can be used in place of the dust of the original Kundt's tube experiment, and the mean distance between the striations of smoke on the wall of the tube caused by the stationary waves can be found from measurements with a travelling microscope.

If an experiment like the one just described is set up using a quartz oscillator (without necessarily including the smoke), it is found that the anode current in the valve circuit passes through maximum and minimum values as the reflector at the far end of the tube is gradually moved in or

out. This is due to the reaction which the stationary waves in the tube exert on the quartz crystal, the maximum reaction occurring when the incident and reflected waves are in opposite phases at the transmitting face. This condition occurs for positions of the reflector separated by half a wave-length, so that the wave-length may be determined from a graph of the anode current against the position of the reflector. This method of measuring wave-length can be used for ultrasonic waves in both gases and liquids. The speed of propagation in the fluid can be deduced from the wave-length measurements if the frequency of the source is known. This latter can be determined from the dimensions of the crystal.

Some Properties of Ultrasonic Waves.—It is found that, although the speed of propagation of audible sound waves in gases does not depend upon frequency, yet in the ultrasonic region speed increases with increasing frequency. This can be accounted for by a consideration of the factor γ in the formula $\sqrt{\frac{\gamma p}{\rho}}$ for the speed of longitudinal waves. On page 433 in Vol. 2 it is shown that

$$\gamma = \frac{q+2}{q}$$

where q is the total number of degrees of freedom of the gaseous molecule, of which three are always in respect of translation. In monatomic molecules there are no additional degrees of freedom, but in diatomic molecules there are two in respect of rotation and in more complex molecules there are more. It appears that when the frequency of vibration of the gaseous molecules due to the ultrasonic waves becomes very high, the changes of pressure occur so rapidly that the heat generated in a compression does not have time to enhance the rotational motion of the molecules before rarefaction occurs and heat is extracted from the region. Thus q tends to diminish to the value 3, so that γ approaches the limiting value of 5/3.

When a plane stationary wave of ultrasonic frequency is set up in a liquid, a structure is produced in which the density of the liquid varies from layer to layer along the direction of propagation of the waves. This structure can be used to diffract light in the same way as the structure of a crystal diffracts X-rays, and measurements of the wave-length of the ultrasonic waves can be made in this way.

Intense ultrasonic radiation is found to have a disruptive effect on liquids by causing bubbles to be formed. The process, which is known as **cavitation,** is similar to the production of bubbles in a liquid behind a fast-moving object. Stable emulsions (*i.e.* suspensions of minute drops) of one liquid in another can often be made by passing ultrasonic waves into the mixture when ordinary mechanical mixing is not successful.

Because of the small wave-length of ultrasonics it is possible to produce a beam of ultrasonic radiation (analogous to a beam of light) which does not spread out laterally by diffraction (see page 591). For this reason ultrasonic waves have been used in determining the depth of the ocean by echo sounding (page 577).

It has been shown by experiment that some animals can directly detect ultrasonic waves, whereas human beings cannot (by definition). Of particular interest is the discovery that bats navigate at night with the help of an "ultrasonic radar" system. That is to say, a bat emits pulses of ultrasonic waves during flight (and at shorter intervals when it is near obstacles) and receives their reflections from trees, buildings, etc., thereby judging the distance and the direction of the obstacles.

EXAMPLES XXXIX

1. Give an account of the possible modes of transverse vibration of a sonometer wire, and show how the frequencies are related to the frequency of the fundamental.

How would you cause a sonometer wire to sound its second overtone, and how would you verify experimentally that it was doing so?

If the second overtone of a sonometer wire of fixed length when under tension T has the same frequency as the first overtone of the same wire under tension T', how are T and T' related? (L.Med.)

2. Explain the meaning of the following terms used in the study of sound: *amplitude*, *pitch*, *timbre* (or *quality*), *overtone*, *beat*, *interval*.

Two stretched wires when plucked have fundamental frequencies of 256 and 320. What is the interval between them, and if they were sounded together what overtones would be in unison? (L.Med.)

3. Explain the terms *overtone*, *harmonic*, illustrating your answer by reference to the vibrations of strings and tuning-forks.

Two strings, A and B, stretched on a sonometer are in unison when set into transverse vibration. The tension on A is increased by 1 kg. wt., and unison is obtained again when the length of B is decreased from 30 cm. to 25 cm. What is the tension in A in the first case? (L.I.)

4. Describe two ways in which a tone-deaf person could determine when a sonometer wire vibrating transversely and a tuning-fork are vibrating in unison.

A, B and C are three stretched wires. When set into transverse vibration C emits a note an octave above that given by A. B can be brought into unison with C by increasing its tension by 3 kg. wt., and into unison with A by an equal decrease of tension. Find the ratio of the frequency of B to that of A. (L.I.)

5. Calculate the original tension in a sonometer wire if, when the tension is increased by 5 kg. wt., the pitch of the fundamental tone is raised by an interval of 3/2. (L.Med., abridged.)

6. Two wires vibrate transversely in unison. The tension in one wire is increased by 1 per cent. and now, when they vibrate simultaneously, three beats are heard in 2 sec. What was the original frequency of vibration of the two wires? (J.M.B.H.S., abridged.)

7. Explain the formation of beats, and state the relation between the frequency of the beats and the frequencies of the separate notes. When two sonometer wires, A and B, made of the same material and having equal diameters and subjected to equal tensions, are sounded together, their fundamental tones give 3 beats per second. A is 42·5 cm. long and B is 43·0 cm. long. Calculate their frequencies. (L.Med.)

8. A wire of length 2 metres supports a weight. When the weight is immersed in water, it is necessary to shorten the wire by 18 cm. if the characteristic note is to remain the same. Calculate the density of the weight. (L.I., abridged.)

9. Two wires, A and B, of the same length and diameter are tuned in such a way that the fundamental of A is in unison with the first overtone of B. If the tension in A is 8 kg. wt. and that in B is 3 kg. wt., calculate the ratio of the densities of the materials of the two wires. (L.I., abridged.)

10. State the laws of vibration of a stretched string and describe how you would verify them experimentally.

A stretched wire is touched lightly with a feather at a point one-third of its length from one end, and is bowed near that end. It is found to be in unison with a fork of 512 cycles per second. Find the length of the wire, if the mass per centimetre length is 0·015 gm., and the tension is applied by a load of 10 kg. (L.I.)

11. Two identical wires stretched on a sonometer under the same tension of 5 kg. wt. emit notes in unison (frequency 300 vibrations per sec.) when plucked.

One wire has its tension increased by 100 gm. wt., and when the wires are plucked beats are heard. Explain the formation of the beats and calculate the number of beats per second which would be heard. (L.H.S.)

12. Describe how the frequency of an alternating current, such as that from a.c. mains, may be determined with a sonometer. (J.M.B.H.S., abridged.)

13. Write down expressions for the velocity of propagation of (*a*) transverse waves along a stretched string, (*b*) longitudinal waves in a rod. In case (*a*) derive an expression for the fundamental frequency of a sonometer wire.

A portion 40 cm long of a wire under a tension of 8 kg. wt. is in unison with a fork of frequency 256. How may the frequency of the wire be raised to 260 by changes in (*a*) its length, (*b*) its tension? (L.I.)

14. Obtain an expression for the velocity of transverse waves along a stretched string.

Describe Melde's experiment for demonstrating the vibrations of a stretched string. (L.I.)

15. How may the dependence of the frequency of transverse vibration of a stretched string on its tension be investigated by Melde's method?

If, in a Melde's experiment, the number of loops on the string decreases from 5 to 4 when the tension is increased by 45 gm. wt., find the initial value of the tension. (L.I.)

16. A thin horizontal wire which passes over a pulley has one end attached to the prong of a vibrating tuning-fork, while the other end is loaded with weights. For a certain value of these weights the horizontal length of the wire vibrates in six loops. Explain this. How does the number of loops change if the diameter of the wire is halved, other conditions being unaltered? (L.I.)

17. A tuning-fork is maintained electrically, the prongs oscillating vertically. One end of a horizontal string is attached to one of the prongs and the string passes over a pulley near the other end to which a weight is attached. Describe what is observed as the vibrating length of the string is gradually altered.

Explain the state of motion in the string. How can a similar condition be generated by sound waves in the air, and how can the wave-length of such waves be measured? (L.H.S.)

18. Write a short account of resonance, giving examples from more than one branch of Physics.

Calculate the shortest length of tube (*a*) open at one end, and (*b*) open at both ends, which will resound to a fork of frequency 280 cycles per sec. when the velocity of sound is 1120 ft./sec. (L.I.)

19. Give an account of the action of a resonance tube and explain how you would determine the velocity of sound in air at 0° C. using a resonance tube. (L.I.)

20. Distinguish between *progressive* and *stationary* waves. Discuss the formation of stationary waves in air in a tube closed at one end.

Air in a tube closed at one end vibrates in resonance with tuning-forks whose frequencies are 210 and 350 vibrations per sec. when the temperature is 20° C. Explain how this is possible and find the effective length of the tube. Assume that the velocity of sound in air at 0° C. is 33,150 cm. per sec. (L.I.)

21. What is meant by *resonance*?

A vertical tube stands in water and a mark is made on its outer surface. A vibrating tuning-fork of frequency 500 vibrations per sec. is held at the upper end of the tube, which is gradually withdrawn from the water. Resonance occurs first when the mark is 4·3 cm. above the surface and next when it is 37·7 cm. above the surface. Calculate the velocity of sound in air under the conditions of the experiment. What can be said about the distance of the mark from the upper end of the tube? (L.I.)

22. Explain how a resonance tube is used *either* (*a*) to determine the velocity of sound in air, *or* (*b*) to compare the velocity of sound in brass with that in air.

The first and second positions of resonance in a tube closed at one end occur at 16·0 cm. and 49·2 cm. from the open end when a fork of frequency 512 cycles per sec. is used. Where do these positions occur with a fork of frequency 384 cycles per sec.? (L.I.)

23. A uniform resonance tube is closed at one end and the shortest length which resonates to a given tuning-fork is 15·2 cm. The next length is 47·6 cm. Explain this. The end of the tube is then opened. What would you now expect the shortest length resonating to the same fork to be? (L.I.)

24. Assuming that the frequency of transverse vibration of a stretched string is inversely proportional to its length, describe how you would investigate the relation between the frequency and the tension. How would you plot your observations and what result would you expect?

A stretched string and an air column closed at one end both resound to a tuning-fork of frequency 256 vibrations per sec., the vibration being the fundamental in both cases. State briefly the differences between the states of vibration of the string and the air column. What is the next higher frequency to which (*a*) the string, (*b*) the air column, will resound without their dimensions or the tension being altered? (L.H.S.)

25. Distinguish between progressive and stationary waves.

Describe an experiment, based on the production of stationary waves, which shows that sound travels more slowly in carbon dioxide than in air at the same temperature. Show how to calculate the ratio of the velocities of sound in these two gases from the experimental measurements. (J.M.B.H.S.)

26. Explain what is meant by resonance.

Describe how you would tune a stretched string to a note an octave below a given tuning-fork, using a resonance method.

A tuning-fork of frequency 440 $sec.^{-1}$ is held over the open mouth of an adjustable resonance tube of the usual type, and the length of the air column is gradually increased by running water out of the lower end of the tube. Resonance occurs first when the level of the water is 18·0 cm. below the mouth of the tube, and a second time when this distance is 55·5 cm. Calculate the velocity of sound in air, and the end correction for the tube. (Adapted from O.H.S.)

27. How would you determine the velocity of sound in air at 0° C. if no ice were available?

A student holding a fork of frequency 600 per sec. over a resonance tube obtains two successive resonance positions for lengths 41·5 cm. and 69·5 cm. Comment on these readings, and use them to determine the velocity of sound and the end correction of the tube. (L.I.)

28. Explain the action of a resonance tube in terms of stationary waves.
A resonance tube is closed at one end, and the shortest lengths of the air column which resonate to forks of frequencies 330 and 660 c.p.s. are 24·0 and 11·5 cm. respectively. Explain these results and deduce what you can from them. (L.I.)

29. State the factors which determine the velocity of sound through a gas. Describe a laboratory method of measuring the velocity of sound in air. (L.I.)

30. What are the relative advantages and disadvantages of the use of a Kundt's tube and a simple resonance tube for the determination of the speed of sound in air?
In an experiment with a Kundt's tube filled with air, two rods of different materials A and B, whose lengths are in the ratio of 2 to 3, give dust heaps 10 cm. and 12 cm. apart, respectively. Find the ratio of the speeds of sound in the materials A and B. (L.Med.)

31. How would you compare the velocity of sound in wood or brass with that of sound in air?
Describe the apparatus, method of procedure and calculation. (L.H.S.)

32. Describe how you would exhibit experimentally *two* of the following:—

(*a*) Lissajous' figures.
(*b*) Chladni's figures.
(*c*) Kundt's dust-tube figures.

Describe some application of *one* of those which you select. (L.I.)

33. Give a short account of the physical principles underlying the construction and use of a violin and an organ pipe. (L.I.)

34. What are the physical concomitants of changes in (*a*) the pitch, (*b*) the quality, of a note?
Describe with the aid of diagrams the actions that occur in the air of a sounding organ pipe. (L.I.)

35. Distinguish between progressive and stationary waves and show, by diagrams or otherwise, how they are related. Explain what a node is, and show that the distance between adjacent nodes is half a wave-length.
If the frequency of the fundamental of a closed pipe is 256, what is the frequency of the first overtone? (L.I.)

36. Describe and explain, with illustrative diagrams, the possible modes of vibration of air in open and closed pipes.
If the third overtone of an open pipe has the same pitch as the second overtone of a closed pipe, calculate the ratio of the effective lengths of the pipes. (L.Med.)

37. Define the terms *fundamental tone, overtone, harmonic, interval.*
A pipe of length l, open at both ends, is sounding its fundamental tone. Neglecting end corrections, find the length of a pipe, closed at one end, for which this tone is the fifth harmonic (second overtone). What is the interval between the fundamental tones of the two pipes? (L.Med.)

38. State in what respects musical tones differ from one another and indicate the physical causes of such differences. Account for the difference in quality of the sounds produced by (*a*) a tuning-fork, (*b*) an open pipe, (*c*) a closed pipe, all of the same fundamental frequency.

39. Describe the nature of the air vibrations in (*a*) an open, (*b*) a closed organ pipe sounding its fundamental, dealing particularly with the pressure and amplitude variations which take place. How can the existence of such variations be demonstrated?
What is the fundamental frequency at 20° C. of an open pipe of effective length 120 cm., the velocity of sound in air at 0° C. being 332 metres per second? (L.I.)

40. Explain briefly how vibrations are set up in some form of organ pipe.
What should be the frequency of the fundamental note of an open cylindrical flue organ pipe of diameter 6 cm. and length 24·5 cm. if the mouth- and open end-corrections of such a pipe are respectively 2·9 and 0·6 times its radius and the speed of sound in air is 340 metre $sec.^{-1}$? (L.Med.)

41. If the frequency of an organ pipe is 500 cycles per second when room temperature is 20° C., calculate its frequency at 10° C. (L.Med., abridged.)

42. Two organ pipes each give notes of frequency 430 vibrations per sec. when the temperature is 15° C. If one is blown with air at 15° C. and the other with air at 19° C., how many beats will be heard per sec.? (L.I., abridged.)

43. A pop-gun consists of a tube 25 cm. long, closed at one end by a cork and at the other by a tightly fitting piston. The piston is pushed slowly in; when the pressure has risen to $1\frac{1}{2}$ atmospheres the cork is violently blown out. Calculate the frequency of the "pop" caused by its ejection. (Velocity of sound in air =340 metres per sec.; end correction to be ignored.) (L.I.)

44. Two organ pipes are sounding side by side and are found to produce 50 beats per minute. The temperature of the air is 15° C. By how much must the temperature be raised in order that the rate of beating shall be increased to 51 per minute? (L.I., abridged.)

45. Explain the production of beats.
Two organ pipes sounding together produce 5 beats per second. If the pipe of lower pitch is shortened by 6 cm. it still has the lower pitch but there are now 3 beats per second. If instead it is lengthened by 9 cm. there are 7 beats per second. What is the effective length of this pipe? (L.I.)

46. Give an account of Kundt's tube and explain how it may be used to compare (*a*) the velocity of sound in two gases, (*b*) the velocity of sound in two solids. (L.I.)

47. How would you compare (*a*) the velocities of sound in wood and air, and (*b*) the velocities of transverse and of longitudinal disturbances in a stretched wire?
A stretched wire 1 metre long gives a note of frequency 2000 vibrations per second when stroked lengthwise with a resined leather. How is the string vibrating and what is the speed of propagation along it of the vibrations? (L.H.S.)

Chapter XL

ACOUSTICAL MEASUREMENTS AND APPARATUS

1. DETERMINATION OF WAVE-LENGTH

Under this heading we need only recapitulate. Acoustical experiments in which the wave-length in air (or another gas) of the sound emitted by a source is determined directly are

The Resonance Tube (page 653).

Kundt's Tube (page 657).

Interference Methods, such as Quincke's tube (page 584).

It should be mentioned that in a sonometer or other musical instrument depending upon the transverse vibration of strings or wires, the wave-length of the sound *on the string* is known from the length of the string (page 637). This is also true of the longitudinal vibrations of rods (page 644).

2. DETERMINATION OF FREQUENCY

Frequency from λ and c.—The methods of evaluating wave-length mentioned in the previous section, whether by experimental determination or from a knowledge of the dimensions of the vibrator, can be regarded as methods of deducing the frequency (n) provided that the appropriate speed of propagation (c) is known. The three quantities are, of course, related by the equation $c=n\lambda$.

Methods of determining the frequency of a tuning-fork in common use in teaching laboratories include the sonometer and the resonance tube. In the former, the sonometer wire is tuned to be in unison with the fork either by ear, by resonance, or by the elimination of beats. The frequency is then calculated from the sonometer formula (page 635). The resonance-tube method of determining the frequency of a fork is explained on page 654. It is necessary to know the speed of sound in the air of the laboratory unless the experiment is repeated with a fork of known frequency.

Use of a Source of Variable known Frequency.—A very old instrument which comes in this category is the **siren.** This consists of a small metal wind chest with a flat top in which a number of equally-spaced holes are drilled round the circumference of a circle. A metal wheel which is free to rotate lies parallel and close to the top of the wind chest and has a similar set of holes drilled in it. The holes in the wind chest are inclined to the plane of the top, and those in the wheel are similarly

inclined, but in the opposite direction (Fig. 477). The wheel is thus caused to rotate by the slanting jets of air which issue from the wind chest. As the wheel spins, a puff of air passes through the holes every time the two sets of holes are opposite to each other, and this gives rise to a sound of frequency equal to the number of times the holes coincide per second. Thus the frequency is equal to the number of holes in the wheel multiplied by the number of revolutions which the wheel performs in one second, which can be obtained by taking at known intervals two or more readings of a revolution counter attached to the spindle of the wheel. In order to find the frequency of a musical note, the rate of blowing into the wind chest is altered (thus altering the speed of the wheel) until the note emitted by the siren is judged to be in unison with the unknown note. The speed is then held as steady as possible, and readings of the revolution counter are taken at the beginning and end of a measured interval of time. The experiment is evidently very inaccurate unless a steady and finely adjustable wind supply is available.

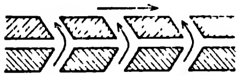


FIG. 477

A very convenient instrument for determining frequency by tuning to unison is a valve oscillator of variable frequency. Such an oscillator often consists of two circuits, each of which produces high frequency electrical oscillations. The two alternating currents are fed to a loudspeaker, and the frequency of one of them is adjusted by means of a variable condenser in its circuit until the audio-frequency beat (or heterodyne) tone (of frequency equal to the difference of the two separate high frequencies) is in unison with the note of unknown frequency. If the dial of the variable condenser has been calibrated against the frequency of the beat tone the frequency of the unknown source can be read off.

The Stroboscope.—The general principle of the stroboscopic method of determining the frequency of a vibrating body is to produce light flashes of a variable known frequency and to view the vibrator in this intermittent light. If the vibrator (*e.g.* a tuning-fork or a piece of reciprocating machinery, etc.) is illuminated only when it is at one particular point in its oscillation, it will appear to stand still in that position, and there is then a simple relationship between its frequency and that of the interrupted light. The simplest case evidently occurs when the two frequencies are equal, but the vibrator also appears stationary in one position if it is illuminated every alternate time it passes through that position, or every third time, or fourth time, etc. These cases occur when the frequency of the light is $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc. of that of the vibrator. If the frequency of the light is raised to double that of the vibrator, the latter will be seen in two positions of its oscillation separated by half a vibration. Similarly it can be made to appear in 3, 4, etc. positions when the light frequency is 3, 4, etc. times the vibrator frequency. Since the appearance of the vibrator is the same whether the frequency of the light is equal to

or a submultiple of that of the vibrator, it is necessary to distinguish the various cases by means of the criterion that as the frequency of the light is raised, the equality case is the last one for which the vibrator is seen in one position only.

It should be mentioned that owing to the fact that our brains continue to register a response for an appreciable fraction of a second after a visual stimulus has been cut off (persistence of vision), there is no impression of flicker in the vibrator which is viewed stroboscopically, unless the frequency of the light is below about 16 per sec. It is this phenomenon which prevents the sensation of flicker in motion pictures. It is also interesting to note that if the frequency of the light is changed a little from one of the frequencies which gives the vibrator a stationary appearance, the latter will appear to vibrate slowly. This is a useful way of examining the motion performed by rotating or reciprocating machinery.

Outfits for the stroboscopic examination of moving objects are manufactured commercially. These consist of a valve oscillator which operates a gas discharge lamp, making it emit flashes of very short duration at a controllable frequency. The shortness of the flashes obviously increases the sharpness with which the object is seen. With such an apparatus it is quite possible to read writing on a rapidly rotating cardboard disc. If the appliance has been calibrated it can be used to determine the frequency of any vibrator within its range of frequency.

A simple stroboscope can be set up in the laboratory by making equally spaced holes or radial slots in a disc of cardboard or thin metal and mounting the disc on the axle of an electric motor whose speed can be varied. The disc may then be interposed between the eye and the vibrating object and the latter viewed through the moving holes. Alternatively the holes may be used to interrupt a strong beam of light incident on the vibrator. In either case, the speed of the motor is varied until the maximum speed for the stationary condition is reached, the vibrator being seen only in one position. The number of revolutions performed by the disc per second is then deduced from readings of a revolution counter taken at known intervals while the speed is kept constant. The number of revolutions per second multiplied by the number of holes in the disc is equal to the frequency of the vibrator in c.p.s.

Interrupted light for stroboscopic experiments can also be produced by means of a mirror attached to a rotating axle or to a vibrating reed.

The principle of the stroboscope may be used to check the speed of revolution of an axle by attaching to the latter a white disc with a black pattern on it such as that shown in Fig. 478. This is then illuminated by an interrupted light source. The number of marks in the pattern is

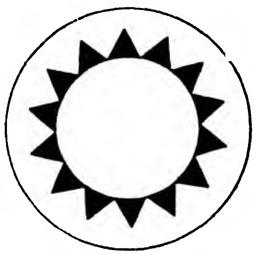


Fig. 478

chosen so as to be appropriate to the frequency of the light source and the speed at which it is desired to maintain the rotating disc. Evidently if conditions are such that each mark of the pattern moves round to the position of its neighbour in the time interval between two flashes, then the pattern appears to stand still. If the lamp is flashing n times per sec., then n marks on the apparently stationary pattern must pass a given point in one second. Therefore if there are m marks in the pattern the number of revolutions per second is n/m.

An ordinary metal filament lamp operated by an alternating current supply will produce stroboscopic effects which can be simply demonstrated by moving a bright object about near the lamp. The object appears to move in a succession of jerks instead of continuously. The light from a lamp running on, say, 50 cycle a.c. fluctuates between a maximum and minimum 100 times per second, because the emission reaches a maximum twice during each complete a.c. cycle—once for each maximum value of the current irrespective of direction. A filament lamp does not give very sharp stroboscopic effects, because the duration of low illumination is only a small fraction of the period of oscillation. Gas discharge lamps are much more effective in this connection. Discs carrying a ring of equally spaced marks are sold for placing on gramophone turntables in order to adjust the speed of rotation to the correct value, the light from an ordinary lamp operated from the 50 cycle a.c. mains being the illuminant. The reader can verify that the whole number of marks on the pattern which will allow the speed to be adjusted as nearly as possible to the usual speed of 78 r.p.m. is 77.

The Phonic Motor.—This device is, in effect, a very low-power electric motor whose speed is controlled by an interrupted direct current or by an alternating current. Its simplest form is shown in Fig. 479, in which A is the rotor consisting of a thick cogged wheel made of soft iron, BB are two coils of insulated wire wound round soft iron cores and are placed diametrically opposite to each other as shown in the drawing. The phonic motor is controlled by an electrically maintained tuning-fork C which operates in the following way. An electromagnet D is situated between the prongs of the fork and, as the diagram shows, the current passing through it also passes between two platinum contacts E, one of which is fixed and the other is attached to one prong of the fork. The electrical power is derived from three or four accumulators in series. If the contacts are properly adjusted and the fork is set into vibration by pressing its prongs together and releasing them, it will continue to vibrate with a constant amplitude, because each time the prongs move outwards,

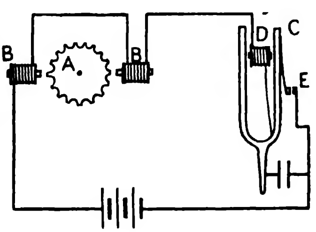


FIG. 479

contact is made at E, so that the magnet D is energized and the prongs are pulled together. The contact is then automatically broken, the magnet ceases to attract the prongs and they are allowed to swing back until the contacts again touch. The operation of the fork evidently causes the electromagnets BB of the phonic motor to be energized once during each vibration of the fork, namely each time the contacts touch each other. Therefore if the wheel is rotated by hand at an increasing rate until the number of cogs passing each of the magnets per second is equal to the frequency of the fork, it will continue to turn steadily at this speed, each cog in turn receiving a forward impulse just before it reaches the magnetic pole. If the wheel is speeded up by hand above this steady rate it experiences a retarding force, due to the fact that the cogs are attracted to the poles *after* they have passed them. A flywheel mounted on the same axle will help to keep the motion smooth and steady, and a fairly large electrical condenser should be connected across the contacts E as shown so as to minimize sparking.

Evidently the number of vibrations performed by the fork in a given time is equal to the product of the number of revolutions made by the wheel in that time by the number of cogs on the wheel. Alternating current mains, with the voltage suitably stepped down to 10 or 12 volts, can be used to run the phonic motor instead of the tuning-fork. In this case the frequency with which cogs pass the poles when the motor is running at the correct speed is double that of the a.c., because the magnets are energized twice during each cycle and there is an attraction between the poles and the cogs irrespective of the direction of the current. This principle is used in electric clocks.

Beats. Lissajous' Figures.—The phenomenon of beats is discussed on page 585, where it is shown that when two sound waves of approximately equal amplitude and slightly differing frequency are arriving at the ear (or some other sound detector), the result is a periodic waxing and waning of the sound. The frequency of these beats is equal to the difference of the frequencies of the separate notes. It is therefore possible to determine an unknown frequency by counting the number of beats per second when it and a source of standard frequency, such as a tuning-fork, are sounded together. It is explained on page 587 how to decide which of the two sources has the higher frequency if this cannot be judged by ear. We shall now describe another way in which frequency difference can be examined and measured. It is concerned with the addition (or "composition") of two simple harmonic motions which are taking place in directions at right angles to each other.

We shall consider by a graphical method the resultant path described by a particle which is performing simultaneously two S.H.M.s at right angles to each other, namely a horizontal motion along OX (Fig. 480) with an amplitude equal to half OX and a time period T, and a vertical motion with an amplitude of half OY and a different period which we

have chosen to be $\frac{4}{3}T$. A semicircle is constructed on OX and its arc is divided into, say, six equal portions, each of which will represent the same interval of time because S.H.M. can be regarded as being generated by a point moving with uniform speed round the circumference of a circle. The choice of the number six makes it possible to fulfil the condition that the period of the vertical motion is $\frac{4}{3}T$ by drawing a semicircle on OY and dividing its arc into eight equal parts and supposing that each of these represents the same time interval as each sixth part of the other semicircle. From each of the equidistant points on the arcs we drop perpendiculars on to the corresponding diameters OX and OY and continue them into the rectangle contained by OX and OY. Suppose that the particle whose path we are going to trace is at O at a certain instant. Then, starting from O, the horizontal motion would take the particle to A in the same time as the vertical motion would take it to F. The actual position of the particle after this time, therefore, is the point of intersection of the vertical line through A with the horizontal line through F. After the next interval the particle will be situated at the intersection of the lines through B and G and so on. Thus the path of the particle (known as a **Lissajous' figure**) can be traced as shown in the figure by proceeding from the point of intersection of one pair of lines to the point of intersection of the next pair and so on. When the point Y is reached, the particle retraces its original path back to O, thus completing a cycle. It can be seen from the figure that during this cycle the S.H.M. with the period T (*i.e.* the horizontal motion) has completed four oscillations, while the slower (vertical) motion of period $\frac{4}{3}T$ has completed only three vibrations, the time taken being $4T$ in each case. In general, if one of the time periods is T and the other is $\frac{p}{q} \cdot T$ where p and q are whole numbers and p is greater than q, then the time necessary for the completion of the cycle is pT, during which the S.H.M. with the period T performs p complete oscillations and the other performs q oscillations. Thus when p or q, or both, are large numbers, so that p/q is not a simple ratio like $\frac{2}{3}$, $\frac{4}{3}$, etc., the pattern is more complex. It is evidently impossible for the pattern to be exactly completed and repeated unless p and q are whole numbers, because the necessary condition for completion is that each motion shall perform a whole number of oscillations during each cycle.

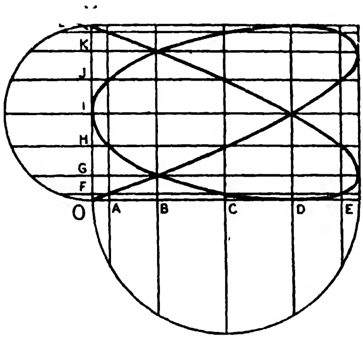


FIG. 480

The amplitudes of the separate vibrations have been chosen to be in the ratio of 3 : 2 in the drawing, but the actual value of this ratio only

affects the relative vertical and horizontal dimensions of the pattern. The character of the pattern is determined by the ratio of the time periods and by a phase relationship between the two vibrations. It is sometimes stated that for a given frequency ratio, the pattern is decided by the phase difference between the two vibrations. This is an insufficiently exact statement because, as we have seen, the phase difference changes continuously during the tracing of a complete pattern. The shape of the trace in Fig. 480 was decided by our supposing that at a certain time both vibrations caused the particle to be at O. Other patterns would result from supposing that when the vertical motion has brought the particle down to OX the horizontal motion places it at various points in OX other than O. It must also be remembered that from each of these positions the particle may move off either to the right or to the left.

Fig. 481 is one of the other possible patterns obtainable from a time-

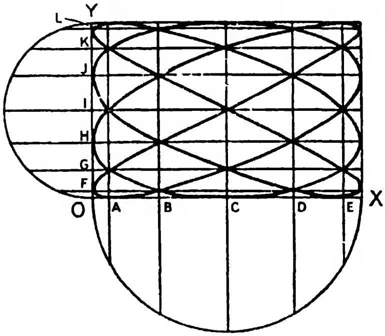


Fig. 481

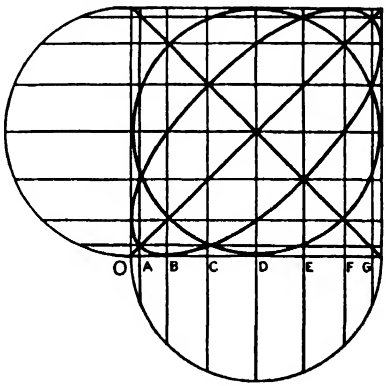


Fig. 482

period ratio of 4 : 3. It can be traced by starting from A and moving to the right. It will be noticed that the particle does not retrace any of its path in the reverse direction as in Fig. 480 but moves continuously along a closed curve.

It is clear that the formation of Lissajous' figures by two mutually perpendicular S.H.M.s provides a very sensitive method of determining or checking the ratio of the frequencies of the two vibrations, especially if this is a ratio of two small numbers. If a pattern is formed which is repeated time after time without variation, the ratio of the frequencies must be expressible in terms of two whole numbers, each of which can be found from the number of times the trace crosses the enclosing rectangle and returns during the performance of the complete cycle.

As a special case we may mention the figures produced when the frequencies and the amplitudes of the motions are exactly equal. The construction is shown in Fig. 482, and it will be seen that by starting at the

point O the trace is simply a straight line at an angle of 45° to the directions of the two motions. Commencing at A, B or C gives an ellipse, D gives a circle, E, F and G other ellipses and X a straight line. In the particular case of equal frequencies the phase difference between the two vibrations is a constant for any given cycle. Typical patterns for phase differences of zero (passing through O), $\pi/4$ (passing through B), $\pi/2$ (passing through D), $3\pi/4$ (passing through F) and π (passing through X) are shown in Fig. 482.

An important case arises when the frequency ratio is slightly different from the ratio of two small numbers. For simplicity we shall consider the case of two S.H.M.s, denoted respectively by a and b, whose frequencies are respectively $(n+p)$ and n, where p is small compared with n. The frequency ratio is therefore nearly 1 : 1. We can discuss the resultant motion as follows. In Fig. 483 the faster oscillation, a, is supposed to take place along the horizontal line OX and b along OY. Suppose that we start from O. Then if the ratio had been exactly 1 : 1 the pattern would have been the straight line OP. As it is, with a moving slightly faster than b, the edge PX is reached at some point A below P instead of at P. When the b oscillation has reached the top edge YP, the a oscillation has started to move back from X so that the point B is reached, then C, D and so on, each point getting slightly further from the corner of the square. Thus the pattern starts with almost a straight line and develops into an ever widening loop, which eventually becomes almost a circle when the phase of a has become $\pi/2$ ahead of b. Thus with a's phase lead increasing steadily, the resultant motion passes through stages closely resembling those characteristic of a 1 : 1 ratio of frequencies (Fig. 482) until the line YX is reached, when the phase lead has become π. The shapes in Fig. 482 are then passed through in the reverse direction until the trace again closely approximates to the line OP. At this stage a has gained one vibration on b, *i.e.* a phase lead of 2π.

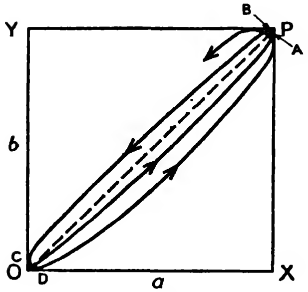


FIG. 483

If the actual frequencies of the vibrations are not too high, it is possible to see the gradual change of pattern and to find the time interval between consecutive occurrences of any one particular pattern. The straight line is the easiest to recognize for this purpose. Suppose that the mean time interval is t sec. In this time a performs $(n+p)t$ vibrations and b does nt vibrations, and, as we have seen, the difference between these two numbers must be one vibration. Thus

$$(n+p)t - nt = 1$$

or

$$p = \frac{1}{t}$$

This principle provides a very accurate method for determining p, *i.e.* the difference between two nearly equal frequencies.

Two tuning-forks may be used for the production of Lissajous' figures, and the difference between their frequencies can be found as follows. The two forks *a* and *b* are firmly mounted at right angles to each other as shown in Fig. 484. A narrow beam of light is directed almost normally on to the polished prong of *b*, from which the light is reflected on to the prong of *a* and thence to a screen where it forms a spot of light. If the prongs of the forks are not or cannot be polished sufficiently to reflect light, small mirrors may be attached to them, but the additional mass will, of course, lower the frequencies of the forks. It is evident from the drawing that the oscillations of *a* cause the spot of light to move vertically up and down on the screen, while those of *b* cause the spot to move horizontally. When both forks are vibrating simultaneously as a result of bowing or striking, the spot is subjected to two mutually perpendicular S.H.M.s and will describe a Lissajous' figure on the screen. Owing to the persistence of vision the path of the spot will appear as a trace of light. If the frequencies of the forks are exactly equal, the figure will be of one of the types shown in Fig. 482, while if they are not quite equal it will change continuously from one type to the next, as already explained. The difference between the frequencies of the forks (which is equal to the frequency with which the Lissajous' figure passes through a complete cycle of change) can be determined by timing with a stop-watch as many repetitions of the complete cycle of change as are observable. To decide which fork has the higher frequency it is necessary to observe the effect on the Lissajous' figures of adding increasing small loads to the prongs of one fork. Deductions are then made as described in connection with beats on page 587.

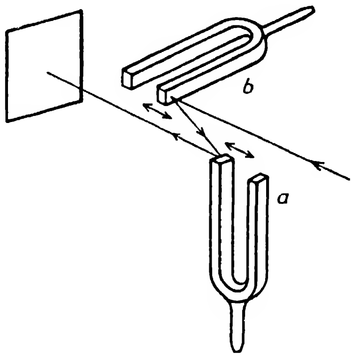


FIG. 484

There are other ways in which two tuning-forks can be made to produce Lissajous' figures, *e.g.* a small powerful lens may be mounted on one prong of one of the forks and a small bright point on a prong of the other fork is observed through it. The bright point may conveniently be a small drop of mercury made to adhere to the fork by grease.

A simple apparatus for demonstrating the tracing of Lissajous' figures is illustrated in Fig. 485. It is known as Blackburn's pendulum. Two long pieces of thread are attached to supports A and B on the same horizontal level and a heavy funnel, F, is attached at their lower ends. The

two threads are brought together at C by a small wire ring which can be moved up and down them. The funnel is filled with sand, so that it can trace out a line on a horizontal sheet of paper placed below it. When the funnel is drawn aside from its position of rest vertically below D, the midpoint of AB, and then released, its subsequent motion is a combination of two mutually perpendicular S.H.M.s. One of these is indicated in the figure by the dotted arc a and is the same as the motion of a simple pendulum of length DF, while the other is the arc b corresponding to a pendulum of length CF. Various frequency ratios can be obtained by altering the position of C. Since the time period of a simple pendulum is proportional to the square root of its length, the ratio of the time periods of the two motions of F is $\sqrt{l_1/l_2}$.

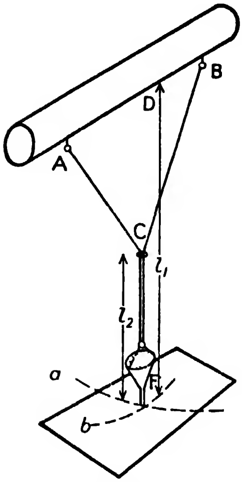


FIG. 485

The instrument *par excellence* for the demonstration of Lissajous' figures is the cathode-ray tube, in which an electron beam which causes a spot on the fluorescent end of the tube can be deflected in two mutually perpendicular directions by electrical potential differences applied to two pairs of deflector plates between which the beam passes. The most common popular use of the cathode-ray tube is, of course, in television sets. The very small inertia of the electron beam allows it to respond faithfully to very high frequency oscillations.

3. MEASUREMENT OF INTENSITY

As explained on page 563, the intensity of a sound at any place is a physical quantity equal to the rate of flow of energy (energy flux) across unit area placed perpendicular to the direction of propagation.

The formula for the energy flux of a wave of amplitude a is given on page 563 as $2\pi^2\rho ca^2n^2$, and it will be seen that it can be evaluated for a note of known frequency if the amplitude is measured. This measurement can be made in a direct manner in the case of a sound wave in a gas by putting light smoke particles in the gas and observing them with a microscope in strong illumination. The rapidly moving smoke particles appear as short luminous lines, and there is reason to suppose that the actual gas molecules perform motions of the same amplitude as the smoke particles, which, of course, can be measured.

The Rayleigh Disc.—An important absolute method of measuring sound intensity is by means of the Rayleigh disc. The action of this device is based on the phenomenon that a flat disc placed in a stream of fluid and free to rotate about a diameter, experiences a couple tending to

make it set with its plane at right angles to the direction of flow. The cause of the action can be seen from Fig. 486 (i), in which the lines of flow of the fluid past the disc are drawn. Where the lines are crowded together the velocity is large and, by Bernoulli's equation (page 207, Vol. 1), the pressure is correspondingly small. Therefore the pressure in the regions marked A in the figure is lower than elsewhere, and a couple is exerted on the disc in a clockwise direction. When the disc is perpendicular to the lines of flow (Fig. 486 (ii)) there is no resultant couple.

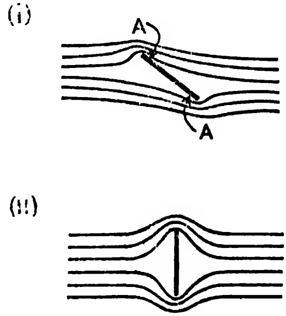


Fig. 486

A discussion of the formula for the magnitude of the couple is beyond the scope of this book, and we need only mention that it is greatest when the disc makes an angle of 45° with the direction of the stream and that it is proportional to the square of the velocity of the stream. The latter fact causes the couple to remain in the same sense when the direction of the stream is reversed, so that the disc tends to rotate into its broadside-on position even when the direction of the flow is alternating as in a longitudinal sound wave. The root mean square particle velocity then replaces the stream velocity in the formula for the couple.

In measurements of sound intensity in gases, the disc is a small piece of thin mica suspended by a fine fibre at an angle of 45° to the direction of the sound (for maximum sensitivity), and its deflection when the sound is present is observed by a lamp and scale. If the torsional constant of the suspension is known, the couple can be calculated and so the *particle* velocity can be found. This can be related to the energy flux.

The Condenser Microphone.—This is a modern method of intensity measurement. When two flat plates are placed parallel to each other (A and B in Fig. 487) they constitute an electrical condenser. In the condenser microphone B is rigidly fixed, while A is a thin circular diaphragm clamped round its edge. A source of direct electromotive force of 100 volts or so is connected across the plates. Any movement of A towards or away from B alters the capacitance of the condenser and causes a transient electric current to flow in the circuit joining the two plates. If a sound wave is incident on A, the consequent oscillations of A cause a small alternating current to flow which can be measured. The magnitude of the alternating current can then be related to the intensity of the sound if a previous calibration has been done in which the current produced is measured when the microphone is subjected to various sound intensities which have been determined by some other method, such as a Rayleigh disc.

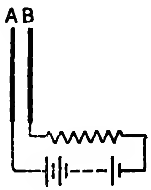


Fig. 487

4. SOUND ANALYSIS

Sound analysis consists in the determination of the actual or relative intensities of the fundamental and overtones present in a sound. As we have often mentioned, the actual oscillation of a particle taking part in a sound vibration, however unlike a sinusoidal motion it may appear to be, can be regarded as a combination of simple harmonic vibrations of frequencies proportional to the numbers 1, 2, 3 . . . and various amplitudes determined by the actual form of the vibrations. The separate components are harmonics, the first harmonic having the frequency of the fundamental tone, *i.e.* the frequency with which the complicated wave-form repeats itself.

Analysis by Ear.—The harmonics present in a sustained note from a musical instrument can be detected by the ear, especially if a note of the same pitch as the harmonic to be identified is sounded previously as a guide. For example, if the C which is one octave above middle C is sounded on a piano and, after it has died away, middle C is sounded loudly, the presence of the octave above (the second harmonic or first overtone of middle C) can be detected in the note given out. The G above this and then the C two octaves above the fundamental and higher harmonics can also be heard in the same way.

Recording Wave-Form.—The relative amplitudes of the harmonics of a musical note may be determined by finding the wave-form and then analysing it mathematically or by a machine known as a **harmonic analyser.** Much pioneer work in the tracing of wave-forms was done by purely mechanical instruments known as **phonodeiks.** In these, a thin diaphragm is placed across the narrow end of a horn into which the sound is passing, and the movements of the diaphragm are made to cause a corresponding rotating movement of a small mirror from which light is reflected on to a moving sheet of photographic paper. It is possible to obtain the necessary movement of the mirror by the simple device of actually attaching it to the diaphragm about half-way between the centre and edge of the latter. In a more sensitive form of the instrument due to Miller, a light thread A (Fig. 488) is attached to the centre of the diaphragm, wrapped round a small drum B and then held taut by a light spring C attached to a fixture E. The vibrations of the diaphragm cause the drum to rotate in a corresponding way, and light reflected from the mirror D, mounted on the same spindle as the drum, will form a trace on moving photographic paper.

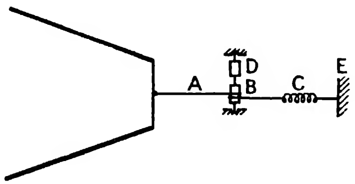


Fig. 488

The diaphragm and its attachments constitute a mechanical system with its own natural period of vibration. The pressure variations which

occur in front of the diaphragm as a result of the incidence of sound waves cause the system to perform forced oscillations according to the principles discussed in Section 2 of Chapter XXXVII. Evidently it is desirable that a given pressure change shall produce the same displacement of the phonodeik trace no matter how slowly or quickly the change occurs. For instance, if the apparatus responds less to quick pressure changes than to slow ones, the finer structure of the record will not be reproduced properly and may even be lost altogether, leaving only the broad outline. It is equally clear by inspection of Fig. 440 (page 612) that uniform response cannot be achieved over an indefinitely wide range of frequency, because, in order to do so, it would be necessary for the graph of the amplitude of the forced vibration against the frequency of the applied force to be a straight line parallel to the frequency axis. To approximate to these ideal conditions it is necessary to choose the properties of the system so that its response is represented by a curve similar to the lower curves in Fig. 440, and to extend the roughly horizontal initial portion as far to the right as possible by causing the system to have a high natural undamped frequency. In order to achieve the latter condition, the mass of the system must be small compared with the restoring force provided by the spring. The small mass and the comparatively large *resisting* force which the air offers to the movements of the diaphragm combine (as explained on page 612) to cause the response-curve to be of the flat type like the lower curves in Fig. 440, which is what is required. It is necessary to remember, in addition, that if the damping is not sufficient to make the system critically damped, then the natural vibrations of the system will be superimposed on those due to the sound waves incident on the diaphragm. Therefore, even though less damping than the critical amount would give a more nearly horizontal curve, the system must, in practice, be at least critically damped, *i.e.* the curve must be something like the lowest one in Fig. 440.

The above considerations apply to all systems, *e.g.* microphones, telephones and loud-speakers, in which it is intended that the instantaneous value of the displacement shall be a true measure of the instantaneous magnitude of the force over a wide frequency range.

The cathode-ray tube, in which a beam of electrons is deflected by an electric or magnetic field, gives a perfect reproduction of the fluctuations of the deflecting field because the mass of the electron beam is practically non-existent. In using a cathode-ray tube to examine the wave-form of sound waves, however, it is necessary to introduce some form of microphone in order to convert the sound vibrations into electrical ones, and the response of this instrument is necessarily imperfect.

When a wave-form has been traced it can, as already mentioned, be analysed by a machine which calculates the relative amplitudes of the harmonics of the particular sound responsible for the trace.

Analysis by Resonance.—An obvious way of analysing a sound of

given frequency is to construct a series of resonators, say of the Helmholtz type, each having a natural frequency equal to one of the harmonics which are to be examined. The response of each resonator when the sound is incident upon it may be detected by placing the ear at a hole in the wall of the cavity, by means of a manometric capsule let into the wall (page 664) or by a hot wire microphone situated in the mouth of the resonator.

The strings of a piano can, of course, be regarded as a series of resonators, although they are comparatively insensitive. If the strings are exposed by removing the front of the instrument and the damper of any one of them is raised by depressing the appropriate key, then when a loud note is played on another instrument or sung near the piano and suddenly stopped, the piano string will be heard to be vibrating if its natural frequency was one of the harmonic constituents of the original note. The harmonics present in a given sound can be picked out by trial and error in this way. Another interesting experiment consists in freeing all the strings by depressing the sustaining (or "loud") pedal and singing loudly a pure vowel sound. The appropriate strings will resonate to the various harmonics to a degree depending on their intensities, with the result that, when the singing is suddenly stopped, the piano is heard "imitating" the sound. It may be mentioned here that vowel sounds are distinguished from each other by their wave-form, *i.e.* by the relative intensities of the harmonics.

Sound Synthesis.—A striking demonstration of the fact that a sound of any quality (*i.e.* wave-form) can be regarded as a mixture of harmonics of appropriate relative amplitudes is the action of electric organs. In the Hammond type of organ, sinusoidal alternating currents of various definite frequencies are generated continuously, and the depression of any one of the keys causes a mixture of these currents corresponding to certain harmonics of the particular note represented by the key to be fed to the loud-speakers. The relative amplitudes of the harmonics can be varied by the player so as to imitate the tone quality of the various stops of a pipe organ.

It is also possible to imitate the quality of various musical instruments by means of a number of tuning-forks whose frequencies form a harmonic series. Each fork gives a pure note without overtones, and different qualities are produced according to which forks are struck or bowed simultaneously and the relative amplitudes of each.

Sound Spectra.—The results of sound analysis are often represented in the way shown in Fig. 489. In this so-called "sound spectrum" the frequencies of the harmonics are marked on the horizontal axis, and an ordinate proportional in height to the amplitude of each particular harmonic is erected at the appropriate frequency. A given class of musical instrument, such as a horn or oboe, gives a certain type of spectrum, but the spectrum may vary between particular instruments in a given class.

There is an especially large variation, for example, from one violin to another on account of the different modes of vibrations of the wooden belly.

Fig. 489 actually represents the theoretical relative amplitudes of the harmonics of a stretched string plucked at a point one-seventh of its length from one end.

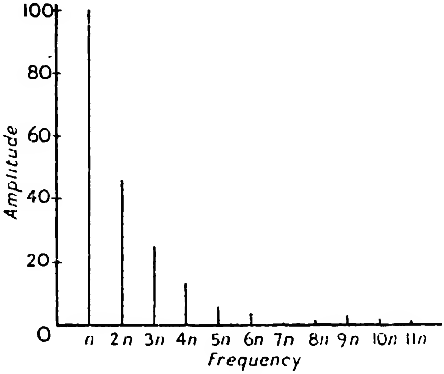


FIG. 489

It is interesting to note that it is quite possible for the amplitude of the fundamental tone (*i.e.* the first harmonic) to be smaller than that of some of the higher harmonics. This is very much the case in the oboe, and it is usually true in the violin. The actual loudness of the fundamental is, it will be remembered, enhanced by the fact that the difference tone between consecutive harmonics has the same pitch as the fundamental (page 626).

EXAMPLES XL

1. Describe in detail a method of determining the frequency of a tuning-fork which does not involve a knowledge of the velocity of sound in air.
Two tuning-forks of frequencies 512 and 516 respectively are sounded together. Describe and account for the phenomena observed. (L.I.)

2. Describe one or two experiments to test each of the following statements:—
   (*a*) If two notes are recognized by ear to be of the same pitch, their sources are making the same number of vibrations per second.
   (*b*) The musical interval between two notes is determined by the ratio of the frequencies of the vibrating sources of the notes. (L.H.S.)

3. How may the frequency of a note be measured experimentally?
A wire of mass 0·1 gm. and length 392 cm. is subjected to a tension of 36 gm. wt. and made to vibrate in four complete loops. Suggest a method by which this could be achieved and calculate the frequency of the note emitted. (L.Med.)

4. Write briefly what you know of the following:—
   (*a*) Beats.
   (*b*) The stroboscope.
   (*c*) The physical basis of musical pitch. (L.I.)

5. Describe an experiment to determine the frequency of a tuning-fork which does not require knowledge of any acoustical data.

What will be heard if two forks of very nearly the same frequency are sounded simultaneously? Prove any quantitative statement you make. (L.I.)

6. Describe, and explain fully, how you would determine the frequency of a tuning-fork by means of (*a*) a resonance tube, (*b*) a stroboscopic method. (L.Med.)

7. A stretched string emits a note of frequency n when plucked. Its tension is increased by 125 per cent. and its length by 20 per cent., and it is again plucked. What is the interval between the original and final note heard?

How could the value of n be found experimentally? (L.Med.)

8. Describe the method and explain the theory of a practical determination of the frequency of a tuning-fork which does not assume the velocity of sound or the frequency of another fork.

If the frequency of a tuning-fork is 550 cycles/sec., where will the first and second resonance positions be located in a resonance tube whose end correction is 3·5 cm.? Assume the velocity of sound in air at 0° C. is 330 metres/sec. and that room temperature is 20° C. (L.A.)

ANSWERS TO EXAMPLES

Examples XXXIV. Page 568

4. $y = 2a \sin \frac{2\pi t}{T} . \cos \frac{2\pi x}{\lambda}$, the wall being at $x = \lambda/4$.
5. (*a*) 740 c.p.s., (*b*) 1480 c.p.s.
6. (i) 5×10^{-6} cm., (ii) 85 cm., (iii) 72°.

Examples XXXV. Page 580

1. 53° 6′.
2. 304 m. sec.$^{-1}$.
3. 57·3° C.
4. 33,310 cm. sec.$^{-1}$.
5. 350 m. sec.$^{-1}$.
6. 1·39.
7. (*a*) 331 m. sec.$^{-1}$, (*b*) 20·2° C.
8. 336 m. sec.$^{-1}$, 39° C.
9. 28,020 cm. sec.$^{-1}$
10. 68·2 m.p h.
11. 217 c.p.s., 185 c.p.s.
13. 27 m.p.h.
14. (*a*) 70 ft. sec.$^{-1}$ towards observer, (*b*) 75 ft. sec.$^{-1}$ away from observer.
15. $n = n_0 \left\{ 1 - \frac{v^2 t}{V(v^2t^2 + l^2)^{\frac{1}{2}}} \right\}$, where t is measured from the instant when the train is opposite the observer.

Examples XXXVI. Page 600

1. (*a*) 1120 ft. sec.$^{-1}$, (*b*) 653 yd
2. 564 c.p.s.
4. 128, 132, 136 . . . 256 c.p s.

Examples XXXVII. Page 620

2. 0·6 cm.

Examples XXXVIII. Page 630

1. 453 c.p.s., 509 c.p.s.
2. 86 db.
3. 59·7.
4. 72 db.

Examples XXXIX. Page 670

1. $T'/T = 9/4$.
2. 5/4 (major third), 3rd overtone of higher and 4th overtone of lower.
3. $\frac{25}{11}$ kg. wt.
4. $\sqrt{5/2}$.
5. 4 kg. wt.
6. 300 c.p.s.
7. A = 258 c.p.s., B = 255 c.p.s.
8. 5·82 gm. cm.$^{-3}$.
9. A/B = 2/3.
10. 75 cm.
11. 3 beats per sec.
13. (*a*) Shorten by 0·6 cm., (*b*) Increase by 0·25 kg. wt.
15. 80 gm. wt.
16. Halved.
18. (*a*) 1 ft., (*b*) 2 ft. (ignoring end corrections).
20. 123 cm.
21. 334 m. sec.$^{-1}$, less than 12·4 cm.
22. 21·5 cm., 65·8 cm.
23. 30·4 cm.
24. (*a*) 512 c.p.s., (*b*) 768 c.p.s.
26. 330 m. sec.$^{-1}$, 0·75 cm.
27. 336 m. sec.$^{-1}$, 0·5 cm.
28. Velocity of sound = 330 m. sec.$^{-1}$, end correction = 1·0 cm
30. A/B = 4/5.
35. 768 c.p.s.
36. Open/closed = 8/5.
37. $5l/2$, 5/1.
39. 143 c.p.s.
40. 486 c.p.s.
41. 491 c.p.s.
42. 3 beats per sec.
43. 510 c.p.s.
44. 11·6 deg. C.
45. 36 cm.
47. 4×10^5 cm. sec.$^{-1}$.

Examples XL. Page 689

3. 60 c.p.s.
7. 5/4.
8. 12·0 cm., 43·1 cm. from the top.

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